1960

The response of milk production to price: a regional analysis

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THE RESPONSE OF MILK PRODUCTION TO PRICE: A REGIONAL ANALYSIS

by

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A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Major Subject: Agricultural Economics

Approved:

Signature was redacted for privacy.

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1960

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INTRODUCTION

Knowledge of supply response is important at all levels in the agricultural sector of the economy. Information on supply relationships could increase the efficiency of agricultural production and marketing. Farmers with a better understanding of their supply function could organize resources for higher profits. With more knowledge of supply response, economists could provide better outlook information. Marketing organizations could increase efficiency if they could predict future supplies. Public policy administrators need information on supply relationships to create more effective farm programs. Thus, greater knowledge of supply response could be translated into lower food costs for the consumer.

The chronic problem of agricultural surplus emphasizes the need for information on supply response. At the same time improved techniques and amended sources of data enhance the possibility of obtaining useful results through research. Therefore, there has been a steadily increasing tempo of work in this area. Behind this interest in supply response lies man's inherent desire to understand and control his environment. Knowledge of how people will respond to changes in economic conditions will make it possible to direct production adjustment for the benefit of society.

Previous Work in Supply

Economists have frequently noted the paucity of empirical work in agricultural supply. This lack of previous research may be attributed to the following: first, research workers have traditionally dealt with normative rather than predictive problems. They have been concerned with questions of "what farmers ought to do" rather than "what farmers are likely to do." Secondly, much of the data needed to carry on predictive research has not been available. Sources of information prior to World War II were extremely limited. Errors of observation in recorded data were in many cases very large. Finally, and most important, too little was understood about farmers' response and factors which affect this response. This lack of understanding has left considerable margin for specification bias in the construction of models.

Production economists over the past half century have employed a number of analytical techniques to better comprehend supply response. Unit cost studies, historical time series analysis, budgeting, and linear programming have been methods used for this purpose. More recently another approach, "the producer panel" has been used. The introduction of this procedure emphasized the fact that much of the research effort must still be concentrated on identifying those factors which affect production response at the farm level.

The techniques which have been used most extensively in supply analysis are budgeting and time series analysis. Farm management specialists have used budgeting (and more recently linear programming) in the analysis of individual farm units and "representative" farm situations. Price analysts have computed regressions using time series data aggregated for the United States. This difference in focus (i.e., micro vs. macro-analysis) has made it impossible to compare results obtained with these techniques. The level of aggregation rather than the nature of the problem has tended to dictate the technique used.

The Scope of the Thesis

This thesis is concerned with supply response in milk production. Regression analysis of time series data was employed to provide regional estimates of supply relations. Empirical estimates were obtained for three major milk producing areas in the United States - the Lake States, the Northeast, and California.

Past studies based on time series data have produced widely varying results. Hence, some economists are critical of this approach. However, there is still much that can be learned, not only through the regression analysis of time series, but also through the graphic examination of these series.

There are two major divisions in the thesis. The first five sections provide a framework for the empirical analysis which follows. A statement of the problem and the objectives is followed by a discussion of previous empirical work on milk supply and related theoretical and statistical problems. The description of the regions studied assists in the selection of the relevant variables for each area. The fifth section sets forth the empirical models used in the analysis.

Results of regression analysis are presented for each region in the following three sections. A short summary of results appears at the end of each section. A fourth empirical section compares estimates of milk supply elasticities for the three regions and for the United States. This section is followed by the summary and conclusions.

This study by providing much needed estimates of regional supply response represents a step toward the solution of the dairy adjustment problem.

THE PROBLEM

Technological innovation has increased the physical productivity of factors throughout the dairy industry. Advances have been made on the farm in crop and livestock production. There have been changes in marketing; improvements in the handling, processing, preservation, and distribution of dairy products. Fewer resources are needed to meet demands.

Surplus milk production has been accompanied by low factor returns during this period of rapid technological change. Production has exceeded demand in all but a few of the post-war years. Income has been lower than for most other types of farming.[a] The cost-price squeeze has emphasized the need for higher factor returns, but farm adjustments have been too slow to keep pace with rising costs.

The decline in per capita consumption of dairy products in the immediate post-war period has increased the adjustment problem. Per capita consumption fell rapidly until 1953 due to the decline in demand for manufactured products, particularly butter. Demand for fluid milk reached a peak during the war and has remained high since then.

Dairy farmers have felt the need for higher incomes. They have attempted to solve their adjustment problems through the market mechanism. Bargaining power has been increased

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[a]See for example U. S. Department of Agriculture, Agricultural Research Service. Farm costs and returns. Agriculture Information Bulletin 176. 1959. Comparisons are given for incomes from different types of farming.

through cooperatives. Federal marketing orders providing for the formula pricing of milk have been adopted in many of the fluid milk sheds. Growing concern with the persistent increase in milk production is further illustrated by recent efforts of dairymen to obtain so called "self help" legislation permitting marketing controls.

Marketing legislation adopted without adequate knowledge of farm response can result in serious resource misallocation. Possible evidence of misallocation is already apparent in the formula pricing of the Northeastern sheds. The New York milk marketing order is a case in point. This order was adopted in 1939. During the first seven years of operation the New York blend price exceeded that of Chicago by approximately seven percent. This margin has increased in recent years to about 30 percent. Vial (75, p. 1) states that the increased spread between the prices in the two markets has been "phenomenal and alarming." From 1940 to 1957 the percentage of milk sold as Class III (principally manufactured milk) has risen from 39 to 45 percent.[a] The short-run effect on supply has not been serious. However, concern about the long-run consequences has encouraged the New York milk marketing administrator to invest considerable funds in research on supply response.

The path of adjustment in dairying should depend upon

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[a]Data after July, 1957 are not comparable because of the marketing area extension effective August 1, 1957.

the goals of society and of the dairy farmer. It would be impossible to develop a set of objectives agreeable to everyone. Conflicting goals lead to uncertainty regarding: (1) the farms and regions which should carry on production, (2) the techniques that should be employed, and (3) the resources that should be retained in production. Conflict of this sort is evidenced, for example, in the controversial health and sanitation requirements. If major goals can be agreed upon, policy decisions to guide farm production must follow. There can be no effective policy without a more adequate understanding of farm response to price and non-price factors.

OBJECTIVES

This study was undertaken with two principal goals: (1) to extend present knowledge of supply response, and (2) to complement the work currently underway in supply response in the Lake States using linear programming. Emphasis has been placed on regional analysis using regression techniques for time series data.

A dairy adjustment study was initiated in the Lake States in the fall of 1959. (The U. S. Department of Agriculture and universities in Illinois, Iowa, Michigan, Minnesota, and Wisconsin have cooperated in this project.) The area defined included most of the three Lake States, plus portions of northeastern Iowa and northern Illinois. The first objective was to determine the regional supply response for milk using the variable-price, linear programming technique.

The results of this thesis will complement the research in the Lake States. Prior analyses using regressions and budgeting or linear programming have seldom been compared. Comparison of estimates using both techniques should (1) improve understanding of supply response in the region, (2) provide more adequate information for studies in interregional competition, and (3) provide a basis for more accurate predictions of supply. These studies by providing regional estimates represent a beginning in the attempt to bridge the the gap between results obtained at the farm level and the highly aggregated results for the United States.

Although the linear programming project has thus far been confined to the Lake States, regression analysis is used in this study in three of the major milk producing areas of the United States: the Lake States, the Northeast, and California. A much less extensive analysis was undertaken for the United States. This permitted a comparison of regional elasticities of supply with those obtained for the entire United States.

The three regions studied account for more than 50 percent of the nation's milk supply. The organization of production, the market outlets (manufactured vs. fluid) and the alternatives to dairying vary among these regions. Therefore, the variables in the analysis also differ among regions. These variables are primarily the price of milk, the prices of competing products, and the costs of inputs.

Short-run estimates of supply elasticity were obtained in each region using several models to determine whether elasticities estimated through different procedures were consistent and reliable. (The short-run has been defined as a period of adjustment extending up to three years; i.e., variables in the analysis are included with lags up to two years.) For example, elasticities were obtained with observations of variables as ratios (e.g., the milk-feed price ratio) and with prices deflated (e.g., the milk price deflated by the index of prices received).

Several hypotheses concerning the elasticity of supply for milk were tested. However, before setting forth these hypotheses the concept of the elasticity of supply, which is used throughout the thesis, is defined.

The elasticity of supply is the percentage change in the quantity divided by the percentage change in the price. The elasticity of supply for milk measures the degree of responsiveness of dairy farmers to a change in the price of milk. The elasticity of supply is related to the slope of the supply function. The supply function is traditionally expressed as a curve showing the relation between the price of the commodity and the quantity offered for sale. According to static economic theory, the short-run supply function is that portion of the marginal unit cost curve lying above the average variable cost curve. The shape of the firm marginal cost curve depends directly on the shape of the production function. Thus a change in the production function can mean a change in the elasticity of supply.

The hypotheses set forth concerning the elasticities of supply were formulated on the basis of the descriptive analysis of the regions studied. They are listed below, but are elaborated in a subsequent section.

(1) The short-run elasticity of supply is higher for the United States than for any of the regions analyzed. Movement in and out of dairying is probably greater in the non-dairy areas where competition from other farm alternatives is strong and fewer fixed resources are

committed to dairying.

(2) The elasticity of supply is greater in the Lake States than in the other two regions. Wider alternative uses exist for feed and less fixed capital is required in the production of manufactured milk.

(3) The elasticity of supply is higher in California than in the Northeast because farm operators in California have a greater economic awareness and there are profitable alternative uses for irrigated land.

(4) The elasticity of supply has risen in the postwar period in the fluid milk sheds (the Northeast and California). Under administered pricing fluid milk producers have experienced greater certainty of price expectations.

The hypothesis that supply elasticities are higher under rising than under falling prices has also been tested.

Regional estimates of supply response from the Lake States are not yet available. The procedure used in obtaining these estimates was as follows. Farms were divided by selected characteristics (e.g., soil type or size according to crop acres) into "homogeneous" categories. The supply response was then to be determined for each homogeneous group using variable price programming. These results would be aggregated to provide a regional estimate of supply response. The linear programming model was designed assuming an adjustment period of approximately five years. Input-output coefficients were based upon university experimental data.

It is believed that, depending somewhat on the procedure used for aggregation, the linear programming results will overestimate the true price elasticity for this period. This is due in part to the normative, profit-maximization assumption of the programming model. The results will therefore set an upper limit to the estimates of price elasticity.

Since the period allowed for adjustment in the regression models of this thesis does not exceed three years, results obtained will tend to underestimate the true price elasticity. They will therefore set a lower limit. The true predictive supply curve will lie somewhere between these extremes.

Preparatory to the empirical analysis in this study, the three following sections contain a discussion of: (1) previous empirical work and theoretical contributions, (2) the production and pricing patterns in the three regions, and (3) the empirical models to be used. The review of previous empirical work and the discussion of the problems encountered in time series analysis forms a basis for selecting the models used. The description of production in the three regions assists in the definition of variables to be included. Several models are considered, because no single model will explain short-run supply response completely.

PREVIOUS RESEARCH: EMPIRICAL, THEORETICAL, AND STATISTICAL CONSIDERATIONS

This section contains a review of previous research dealing with supply response for agricultural commodities. Emphasis is on empirical work in milk supply and upon the important theoretical discussions and statistical problems related to time series analysis. The section is divided into four parts. A discussion of the early empirical work in milk supply is followed by an examination of the important theoretical problems. The third part reviews recent empirical work. The final subsection deals with the statistical problems which hamper empirical investigations of milk supply.

Early Empirical Work

The theory of supply of agricultural products dates from Alfred Marshall's (42) Principles of Economics. Marshall was one of the first to distinguish between the three time periods market, short-run, and long-run. However, there were neither sufficient data nor adequate statistical tools to permit widespread empirical investigation of supply until after the turn of the century.

In 1917 H. L. Moore (47) initiated empirical work by relating quantities to previous prices for several agricultural commodities. His approach was carried forward by Ezekiel, Elliot, Bean, Henry Schultz, and others. During the 1920's a number of studies were conducted in milk supply with seemingly significant results. The procedure was to relate production to the milk-feed price ratio lagged over varying lengths of time. Correlation coefficients secured in this manner were relatively high. Gans and Ezekiel using data from Richmond, Vermont obtained a coefficient of linear determination of 0.554 for the entire year and 0.790 for the winter months. Rauchenstein in the Twin Cities market obtained a coefficient of 0.530.

Work progressed on other commodities as well. The initiative was taken by the U. S. Department of Agriculture and the University of Minnesota. A large number of studies had been completed by the end of the twenties. However, a comparison of the results showed marked inconsistencies and caused many to question the economic significance of this work.

During the 1930's two of the earlier milk studies were reanalyzed for different periods of time. Cassels, and later Stewart Johnson reworked the Vermont study. Quintus retraced the steps of Rauchenstein at Minnesota. All three found insignificant coefficients of linear determination. These insignificant results conflicted with earlier findings.

Theoretical Considerations

Contradictory empirical results led to a re-examination of the theoretical foundation. Model specification (i.e., specification of the form of the relationship and of the variables to be included) became a primary concern. The failures of the early empirical studies were attributed almost entirely to specification bias. This bias occurs frequently in economic studies because it is not possible to neutralize uncontrolled factors by randomization. From the early problem of "identification" to the recent work on "distributed lags," theoretical discussions have emphasized model specification. The contributions toward improved model specification are discussed in the following pages.

The identification problem

In 1927 Elmer Working (82) introduced the now familiar identification problem. He demonstrated that the "demand" curves of H. L. Moore and others were not those of traditional neo-classical theory. These curves were derived by fitting a regression to observations of price plotted against quantity. They could be useful for prediction provided that the movements of supply and demand were correlated. (Observations of supply and demand have seldom been correlated over very long periods of time.) He suggested, however, that the curves be called by some other name to avoid confusion.

Heady (33) has used the term "mongrel" curve. Cochrane (12) has distinguished between the supply relation or supply function (the neo-classical curve) and the supply response (the mongrel curve). These two concepts of supply can be expressed in a functional relationship as follows:

(1) $Q = f(P \mid X_1, X_2, \ldots X_n)$ supply function

(2) $Q = f(P, X_1, X_2, \ldots X_n)$ supply response

where:

Q = the quantity produced

P = the price of the product represented by the dependent variable Q

$X_1 \ldots X_n$ = variables which explain supply shifts and structural change, e.g., time, prices of competing products, quantities of inputs

Variables to the right of the vertical line in Equation 1 are constants.

Earlier workers had formulated models of the first type (Equation 1) ignoring the presence of other factors (i.e., assuming $X_1, X_2 \ldots X_n = 0$). The second formulation (Equation 2) introduces a different problem. It is not empirically possible to include all variables other than price. Hence the danger exists of attributing to price the results of those factors undergoing change which are not included in the model.

The cobweb theorem

The simultaneous and independent discovery of the cobweb theorem is attributed to three economists: Henry Schultz, Tinbergen, and Ricci. Ezekiel (22) reviewed the theorem in an article written in 1938 listing three conditions for application: (1) where production is completely determined by producer response to price under perfect competition, (2) where the time needed for production requires at least one full period before production can be changed once the plans are made, and (3) where price is set by available supply. If the above conditions are fulfilled the system is not simultaneous, and supply and demand equations can be computed by least squares.

There are three situations of price and quantity fluctuations under the cobweb theorem - the continuous, the convergent, and the divergent case. Assume a price-quantity diagram with price on the vertical axis and quantity on the horizontal axis. In this diagram supply and demand functions intersect at a point of equilibrium. The three cases of flucuation depend upon the relationship of the supply and demand curves to each other.

Under continuous fluctuation the demand curve is the exact reverse of the supply curve (i.e., the slopes are equal with opposite signs). Fluctuations in price and quantity follow a recursive pattern. A price, P_{t-1}, in the previous year, will lead to production of quantity, Q_t, in the current year. The price, P_t, in the current year that will remove this quantity from the market will in turn bring forth a production, Q_{t+1}, in the following year. The price required to remove this quantity, Q_{t+1}, is equal to P_{t-1}, the price in the previous year. Hence, prices and quantities fluctuate continuously at a constant level above and below the equilibrium.

In the divergent case, the absolute slope of the demand function is less than that of the supply function. Fluctuations in price and quantity become successively larger. The response is "explosive" and moves further and further away from the equilibrium. In the convergent case, the absolute slope of the supply function is less than that of the demand function. This "dampening" effect on the cycles results in movement toward the equilibrium.

The relevance of the cobweb theorem (whether the continuous, convergent, or divergent case) is based upon the assumption that producers react only to the price of the previous year. Buchannan (7) and more recently Nerlove (50) have challenged this assumption. Nerlove suggests that introducing the concept of lagged response to a change in price would tend to lessen the possibility of instability or divergence.

A number of empirical studies have been based upon the cobweb theorem, the most recent by Dean and Heady (18) for hogs. Milk production does not meet the second of the conditions set forth above. However, Cochrane (15) felt that there was still a strong basis for the operation of the cobweb theorem in milk. He hypothesized a convergent case based upon empirical investigations which indicated a more elastic demand than supply curve.

Reversibility

Cassels' (9) 1933 article emphasized two points concerning the nature of statistical supply curves: (1) the time character, and (2) the irreversibility. With reference to the first point, there is no one short-run supply curve for a given commodity. A whole fan of curves exists, each becoming more elastic with the increase in time allowed for adjustment. The vertical market curve is at one extreme. The long-run curve, whose slope depends on the nature of economies to scale, is at the other. This elaboration of the Marshallian concept of supply embodies the notion of a lagged adjustment over time, i.e., a "distributed lag."

Cassels conceived of a supply curve kinked at the established level of output. Price could change in either direction from the established level. However, the elasticity of expansion (under a rising price) would exceed the elasticity of contraction (under a falling price). These differing rates of expansion and contraction were related to the relative mobility of fixed inputs in a fluctuating economy. Inputs which are fixed in the short-run are variable in the long-run. Hence, irreversibility was associated by Henry Schultz (55) with long-run demand and supply curves.

D. Gale Johnson (35) emphasized the role of factor supply and factor prices during periods of long-run cyclical fluctuation. He contended that the concomitant movement of factor and product prices tends to stabilize factor-product price relationships. This dampens cyclical fluctuation in production for agriculture as a whole. It follows from this argument that the concept of irreversibility is not important in explaining the aggregate supply response for agriculture.

Glenn Johnson (36) placed great stress on the concept of irreversibility at the farm level. He agreed with other economists that the supply curve of rising prices is more elastic than the supply curve of falling prices.

Little empirical work has been undertaken to confirm this well accepted hypothesis that the supply function is not reversible. Halverson's (30) recent study for milk indicates a curve kinked in the opposite direction of that originally assumed. However, the results are inconclusive.

Single vs. simultaneous equations

Through the pioneering work of the Cowles Commission, simultaneous equation analysis has been added to the list of available statistical techniques. The simultaneous equation approach provided an answer to the identification problem raised earlier by Working (82). Rules were established for identifying the coefficients of the simultaneous model. Equations which were, in the language of the Cowles Commission, "over-identified" could be solved through the limited information method.

The introduction of the simultaneous model, however, raised other issues, both economic and statistical. To date, many of these questions remain unsettled. The economic controversy exists between the proponents on the one hand, of a causal sequence and on the other of mutual determination. Parenthetically, this argument exists in various forms in many fields of science and dates in economics from the Walrasian model.

Wold (81) argues against the simultaneous model on two accounts. First, he questions the causal interpretation of the equations. Secondly, he contends that the structural coefficients cannot be interpreted as ordinary supply and demand elasticities. In lieu of the simultaneous approach, Wold offers a recursive system based upon the concept of a "causal chain."

Fox (28), Hildreth and Jarret (34), and others have apparently not considered Wold's criticism to be crucial. They have interpreted coefficients obtained from single and simultaneous equations in like manner. Durbin,[a] on the other hand, adopts an intermediate position. He agrees with Wold's concept of causation. However, he emphasizes the manner in which most time series are collected and reported. Data are collected periodically, usually monthly, quarterly, or annually, thus obscuring causal chains which are of shorter duration and giving the appearance of simultaniety. Durbin thus accepts the simultaneous method on economic grounds, although he holds statistical reservations.

The statistical controversy is set forth in an article by Christ (11). The Cowles Commission demonstrated conclusively that the unbiased estimate of the coefficient of an endogenous variable (i.e., a variable correlated with the error or residual) could not be obtained by using the single equation, least squares method. Christ acknowledges this but states that estimators for the limited information method are statistically inefficient, particularly with small samples. Consequently, the results obtained by limited information may be further

[a]This view appears in a discussion of an article by Wold, Herman. Causal inference from observational data. Royal Stat. Soc. Jour. 119: 52-53. 1956.

from the true value than those obtained from least squares. It is not known what sample size will provide estimates which are more reliable than the admittedly biased results of least squares.

In view of this unsettled issue it seems advisable, (1) to determine whether or not the model is simultaneous, and (2) to solve simultaneous systems, as Foote (25, p. 67) suggests, using both methods.

Distributed lags

The first work on distributed lags was begun during a period when multiple regression studies were being enthusiastically undertaken by some and seriously questioned by others. The term "distributed lag" originated with Irving Fisher. This concept presupposes that the full effect of a change in the independent variable will be worked out only after some lapse of time. Like the cobweb theorem it is dynamic in the Hicksian sense.

Distributed lags can arise from a number of causes: technological or institutional delays in adjustment, psychological inertia, or lack of knowledge. In accordance with neo-classical theory, a lagged response of this nature implies a supply curve more elastic in the long than in the short run.

The problem of determining the form of the distribution of lag, or what Koyck (40) has termed the "time shape of the reaction," has been handled in the past in one of two ways: (1) by making no assumption as to the shape of the distribution, or (2) by assuming a general form and estimating the parameters in accordance with the assumption. Some of the earlier empirical models of Ezekiel and others in which the milk-feed price ratio was represented by a series of variables lagged through time corresponds closely with the first approach. However, a major obstacle in this procedure is the frequent occurrence of multicollinearity between the variables and their lagged counterparts. It is not statistically possible to estimate the entire distribution if the lag occurs over several time periods.

Nerlove (50) has recently employed a model for distributed lags which he refers to as an explicit dynamic model of producer behavior. This approach permits the derivation of short and long-run elasticities of supply without reference to the shape of the distribution of lag. He distinguishes between (1) a <u>coefficient of expectation</u> related to price uncertainty and (2) a <u>coefficient of adjustment</u> related to technological rigidities. In addition he uses a concept of expected normal price (i.e., the price which farmers expect will prevail in the light of past experience and future expectations). The relation between the expected and realized price is expressed as follows:

$$(3) \qquad P^*_t = P^*_{t-1} + \beta (P_{t-1} - P^*_{t-1}) \qquad 0 < \beta \leq 1$$

where:

P^*_t = expected normal price in time period t

P_{t-1} = realized price lagged one year

β = the coefficient of expectation

The hypothesis expressed by Equation 3 states that this year's expected normal price is a function of last year's expected normal price and last year's realized price. If the coefficient of expectation is equal to one, the terms for last year's expected normal price cancel out. In this case the farmer's estimate of expected normal price is based entirely upon the price realized in the previous time period. This adjustment in a single time period is the assumption of the traditional least squares model.

The hypothesis with respect to the lagged adjustment of output is similar:

(4) $$Y_t - Y_{t-1} = \gamma(Y^*_t - Y_{t-1}) \qquad 0 \leq \gamma < 1$$

where:

Y_t = production in the current year

Y^*_t = expected normal output in the current year

γ = coefficient of adjustment

The adjustment of output is assumed to be proportional to the deviation of last year's realized output from what is felt to be the long-run equilibrium output.

Nerlove derives a single equation which incorporates both the elasticity of expectation and of adjustment. This is accomplished with the two relationships in Equation 3 and 4 plus the additional assumption that expected output is a function of expected price, $Y^*_t = \alpha P^*_t$. Values are substituted into an equation of the form:

(5) $$Y_t = a_0 + a_1 P^{*}_t + u_t$$

The resulting equation is:

(6) $$Y_t = a\beta\gamma P_{t-1} + (1-\beta)+(1-\gamma)\ Y_{t-1} - (1-\beta)(1-\gamma)Y_{t-2} + v_t$$

where:

Y_t = production in the current year

P^{*}_t = expected normal price in the current year

P_t = realized price in the current year

a_1 = a constant

β = coefficient of expectation

γ = coefficient of adjustment

u_t and v_t = random errors

In this equation it is impossible to separate the effects of β and γ which enter into the expression symmetrically. In current empirical work it is customary to assume either β or γ equal to one when using the Nerlove model. This eliminates the final term and along with it some of the already serious problems of multicollinearity. The reduced model with γ equal to one can be written as follows:

(7) $$Y_t = a + b\beta P_{t-1} + (1-\beta)Y_{t-1} + u_t$$

where:

b = coefficient of long-run reaction (the long-run elasticity for computations in logarithms).

P_{t-1} = price in the previous year

Y_{t-1} = output in the previous year

Other variables are defined as for Equation 6.

In Equation 7 the coefficient of lagged realized price indicates the short-run supply response. The constant b expresses the long-run supply reaction and is calculated by dividing the coefficient of lagged price by one minus the coefficient of lagged output. Equation 7 is a modification of the difference equation developed by Koyck (40) shown below:

$$(8) \qquad Y_t - Y_{t-1} = a + b\beta P_{t-1} - \beta Y_{t-1} + u_t$$

The symbols are the same as those used in Equations 6 and 7. The relationship between the models of Koyck and Nerlove and the logic of the computation of long-run supply are presented in detail in Appendix B.

Even using the reduced model in Equation 7 it is difficult to obtain meaningful structural coefficients. Brandow (6) has illustrated the problem of specification bias due to the omission of relevant variables. For example, failure to introduce a trend variable may affect the coefficient of lagged output $(1-\beta)$. The direction of the bias will correspond to the direction of the trend in production. Lagged output tends to assume the role of a trend variable. On the other hand inclusion of trend may introduce multicollinearity. This will occur when the dependent variable shows a trend over time. If the equation were used solely for prediction, this problem would cause no concern.

Alternative to time series

Other methods of analysis have historically played an important role in supply studies. These methods were developed in some cases to circumvent the lack of adequate historical data, and in others to meet objectives which could not be achieved through short-run time series analysis. However, the complementary nature of these various approaches is now generally accepted. While the objectives of the study will under most circumstances initially define the appropriate technique, comparison of results using various techniques may greatly enhance our knowledge of supply.

The cost of production studies represented one of the earliest attempts to analyse supply response. Unit costs were drawn together from a large number of farms to estimate average cost curves. However, the information on supply response derived from these cost curves in many instances proved misleading. This approach has long since been abandoned.

Black and Mighell were among the economists who seriously questioned the validity of the increasingly popular time series approach. Moreover, they felt that the advent of multiple regression was in part responsible for the overemphasis on short-run supply analysis. As early as 1924 Black (4) suggested several important objectives of long-run supply studies. Mighell and Allen (45), writing in 1940, re-emphasized the growing need for long-run analysis. They reviewed the possible techniques for obtaining long-run functions and listed three alternatives: (1) the use of statistical data to obtain short-run supply curves (This combined with "informed judgment" would provide a good estimate of the shape and position of the long-run supply curve.), (2) the derivation of supply functions from experimentally determined production functions, (3) the construction of long-run supply curves through budgetary analysis again combined with informed judgement. The authors favored this latter approach. This is described in detail by Mighell and Black (46).

The advent of linear programming facilitated the derivation of supply curves through this procedure. Studies of milk supply in local areas have been conducted by McPherson and Faris (44) and by Easley (21). The aggregation problem remains a major obstacle to regional analysis. The horizontal addition of supply curves will overestimate supply elasticity. Day (16) has proposed an alternative drawing on the concept of "total elasticities" set forth by Buse (8). A system of equations would take into account the simultaneous supply and demand shifts of other products as well as the one of primary interest.

The "producer panel" is the most recent technique to be proposed using this approach. Farmers are questioned and revisited over a period of time to find out: (1) what adjustments are planned, (2) in the light of these plans and changing conditions what adjustments are taking place. The objective is to isolate those price and non-price variables to which farmers respond. A study is underway at Cornell and one has been proposed at Minnesota using this method. It may be a number of years before meaningful results are achieved.

However, this procedure offers much hope for isolating factors which affect supply response at the farm level.

Recent Empirical Work

Recent empirical investigations in milk supply can be classified according to the dependent variable, into three groups: (1) production per cow, (2) cow numbers, and (3) milk output. The most successful statistical results have been obtained with analysis based entirely upon the short-run changes in production per cow. By contrast, efforts to predict changes in cow numbers have proved frustrating. Bachman (1, p. 32) suggests that an entirely different approach may be required to handle the problem of lagged response in livestock cycles. He mentions the possiblity of inventory analysis in the area.

Production per cow

Investigations of changes in production per cow have been conducted by Brandow (5), Halverson (31), Doak (19), and Kottke (39). Brandow's analysis included four states, New York, Pennsylvania, Wisconsin, and Michigan for the period 1933-51. He calculated a three year centered moving average of production per cow and used the deviations from this average as the dependent variable. Analyses were run for both the summer and winter feeding periods. The independent variables for the summer were pasture conditions, and the March milk-feed price ratio. The elasticity of the milk-feed price ratio in these four states varied from .07 to .15. In the analysis of the winter months May-June rainfall and September milk-feed price ratio were the variables used. The coefficients for May-June rainfall were negative. The elasticity of the milk-feed price ratio varied from .04 to .18, being higher in the Midwest than in the Northeast. Brandow suggested that the years of large hay production may be years of poor quality hay. Guided by the results obtained by the New York market administrator, he used May-June rainfall to reflect the quality of hay in the following winter feed period.

Halvorson's analysis partitioned the United States into six regions. He used the milk-feed price ratio, hay production and cow numbers as the independent variables in predicting year to year changes in production per cow. Like Brandow he found negative signs for hay in the winter months. Elasticities for the milk-feed price ratio ranged from zero to .25 with estimates for the summer season lower than for the winter months. There were, however, no distinguishable regional differences.

Halvorson also considered grain fed per cow as the dependent variable and a function of the milk-feed price ratio and hay production. The objective was to determine a more direct measure of farmer response to changing prices. Elasticities for the milk-feed price ratio were considerably higher in the East North Central and the West North Central than in the other four regions of the country. These two regions include many of the Corn Belt states where the alternative uses for feed are numerous.

The studies conducted by Doak and Kottke considered changes in cow numbers as well as production per cow. Doak's study covered 18 states, and the study by Kottke emphasized the explanation of changes in Connecticut production. The results are not easily compared with the works by Brandow and Halvorson.

<u>Cow numbers</u>

Attempts to explain changes in cow numbers have been less successful. The paucity of published material in this area bears witness to the difficulties encountered. Winter (80) explained changes in cow numbers for Iowa with limited success. Doak (19) found the milk-wheat price ratio in the Great Plains and the milk-tobacco price ratio in certain southern states were significant variables. Brandow (5) presents an illuminating discussion of the problems involved. He states that changes in dairy cow numbers are strongly influenced by the comparative advantage of dairying in different areas under changing economic conditions. Over a number of years dairy cow numbers were importantly influenced by changes in the beef cattle cycle. Prices of dairy products, prices of alternative farm commodities and production costs have played an important role but have not been related in any consistent pattern from region to region. Location of markets, physical characteristics, and physical input-output relationships also have affected dairy cow numbers.

Milk production

Some investigators have felt that the explanation of changes in milk supply could best be accomplished through the individual consideration of the component parts, i.e., production per cow and cow numbers. A number of studies has been conducted, however, in which milk production was considered as the dependent variable. This procedure provides a more direct estimate of the elasticity of milk supply. Cromarty (12), using "limited information," found an elasticity of supply of .212 for the United States during the period 1929-53. Halvorson (30) used the single equation distributed lag model developed by Nerlove for two time periods, 1927-57 and 1941-57. Short-run elasticities for the various models ranged from .128 to .185 for the entire period, and from .180 to .312 for the later years. Halvorson suggests that the higher elasticity in the latter period may be due to the introduction of administered pricing in the fluid sheds.

Cochrane (15, p. 76) states that estimates of milk supply relations where cow numbers are not held constant are as likely to have a negative sign as a positive one. Foote (25, p. 171) maintains that "wrong" signs or magnitudes from statistical analyses may lead to a revamping of the underlying theory but are more likely to indicate the need for a different statistical approach in the analysis. These statements emphasize the importance of the statistical problems involved.

The remainder of this section is devoted to a discussion of statistical problems as related to the time series analysis for milk.

Problems in Time Series Analysis

It is possible to divide a given time series, such as the production of milk, into four distinct elements: (1) a secular trend, (2) cyclical fluctuations, (3) seasonal components, and (4) a remainder. Seasonal variation is customarily removed by moving averages. Secular trend is either "removed" by the use of first differences or "accounted for" by introduction of a trend variable such as time. The analysis is thus centered on the explanation of changes in the dependent variable in terms of cyclical fluctuations and the remainder.

The objective of analysis is to yield a model which will: (1) prove useful in the prediction of output, and (2) provide "meaningful" structural coefficients. Not all change in the dependent variable is predictable. Unpredictable changes may or may not be random. Weather variation, for example, is often cyclical in nature.

The signs and elasticities of coefficients are traditionally considered "meaningful" in the light of the restrictive _ceteris paribus_ assumptions of classical economics. By this definition, the equations which provide the best predictive results may not have meaningful structural coefficients. The structural equation which has questionable predictive value may be useful in furthering knowledge of supply response. In a real sense, however, meaningfulness should be associated with predictive power, for this is the ultimate goal. The introduction of the concept of "total elasticity" by Buse (8) is an attempt to resolve this paradox by dropping the ceteris paribus assumption.

Structural change and secular trend

Learn and Cochrane (41) distinguish between shifts in supply and structural change. Shifts in supply result from changes in the values of variables other than price or quantity. Structural change, on the other hand, results from some force which alters one or more of the parameters or the form of the relationship.[a]

There are three classes of supply shifters, (1) prices of factors, (2) prices of competing products, and (3) structural variables. Since a change in the production function is structural, the production function is included under structural variables. The structural variables which appear most commonly in regression models of supply are general price level deflators and trend. These variables are distinct in that they give rise to both supply shifts and structural change.

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[a]A change in the form of the relationship is structural since it involves a change in the value of the coefficient β_i. For example, introducing a variable X_i may in effect alter the value of β_i from zero. Conversely, deleting a variable may change the value of β_i to zero.

Structural changes in supply include: (1) technology, (2) the number and distribution of farms, (3) the skills found within the industry, (4) the knowledge which farmers possess, and (5) the institutional framework surrounding the industry. Structural change is synonomous with secular change to the degree that these five factors vary gradually over time. Many structural changes occur, however, which are not secular in nature.

The concept of secular trend is related to the concept of growth. Growth in supply is in turn related primarily to technological change. Technological change (a change in the production function) is normally accounted for by introducing time as a trend variable. Although time is linear when actual data are used, it may enter into the system as either a power or an exponential function when the data are converted to logarithms. Alternative measures exist for explaining technological change. Cromarty (12), for example, uses the number of Dairy Herd Improvement Associations.

Although the trend variable is associated principally with technological change, it will account for any change which occurs gradually over time. If price relationships show a trend, then a portion of the elasticity due to changing prices will be incorporated in the trend. This can bias short-run price elasticities only in cases where trend is removed from very short series. In this instance the trend variable may account for a portion of a short-run cyclical flucuation rather than the true secular trend in production.

Nonsecular structural change can often be identified and incorporated into the model. Suits (56) in a study of the watermelon industry employs dummy variables to represent the government cotton program and the war years. Dummy variables of this nature are given a value of zero in the non-applicable years, and a value of one during the years the program or event is in effect. They are thus satisfactory for representing "once and for all" changes.

An alternative to incorporating variables which will explain these changes is to divide up the period over which the analysis is run. Data are commonly analyzed for prewar and postwar years separately and the results compared with those for the entire period. This provides a series of "adequate" length for statistical analysis.

This procedure seems appropriate in dairy analysis for a number of reasons. First, feed prices were held down during the war years and farmers were given incentive payments to encourage production. Secondly, technological change in the postwar years has been far more rapid. Finally, reduction in cow numbers prior to the war usually meant smaller herd size. However, declines in the postwar period are closely associated with a decrease in the number of farms. This is further evidence of structural differences between the two periods. In this study regressions have been computed for the periods 1926-58 and 1947-58.

Economists are fully aware of the shortcomings of procedures for handling structural change, but agree generally that some method must be employed, no matter how crude, to approximate a constant environment and a constant state of the arts.

Statistical measurement of price elasticity in milk supply

A major problem in time series analysis is that of separating the effects of price changes which are considered to be only temporary (those to which farmers respond very little or not at all) from those which are assumed to be permanent. Much of the work of Nerlove (50) on price expectations and distributed lags has been addressed to this problem.

Unfortunately, price response is more complex in dairying than in most other agricultural products. Farmers can respond through changes in cow numbers and in production per cow. Changes in production per cow can occur through improved breeding, but in the short-run are related to changes in feed level and in cow numbers. Feed rations are often varied within two or three months following a change in the milk-feed price ratio. Changes in cow numbers can occur immediately by altering the culling rate. However, a change in cow numbers is more commonly related to the number of replacements. Due to the long gestation and growth period the effect is not felt until two or three years following the initial decision.

Another source of difficulty is the comparative stability of the dairy industry, stability both in price and production.

The stability is due in part to the long production life span of the dairy cow and the heavy fixed investment which discourages "inners and outers." It has been further enhanced by milk marketing controls and legislation (particularly in the fluid sheds). In spite of large seasonal fluctuations, the year to year change in milk production for the United States, 1926-58 has averaged less than 2.0 percent. This is significant in the light of Fox's (27, p. 53) statement that as much as 25 percent of the reported year to year variation in the production of milk may come from errors in measurement. The small amount of variation and large error of observation makes the task of determining reliable structural coefficients extremely difficult.

Error exists in most models both in the variables and in the equation. It is not at present possible to analyze systems which simultaneously assume error from these two sources. It is usually assumed that the error in the equation is more important than errors in observation of the independent variables. This is illustrated in Equation 9 below. The alternative assumption, shown in Equation 9A, can be handled only through the complex computational procedure of weighted regression.

(9) $$Y = a + \Sigma\, b_i X_i + \Sigma\, \varepsilon_i$$

(9A) $$Y = a + \Sigma\, b_i (X_i + \eta_i)$$

where:

Y = the dependent variable

X_i = an independent variable

ε_i = the error in the equation

η_i = the error in the independent variable

a = a constant

b_i = the coefficient of an independent variable

The above comments do not imply that dairy farmers are unresponsive to changes in price, but rather that historical stability and errors of observation make this response difficult to measure accurately. The difficulty exists irrespective of the logical formulation of the model and the appropriateness of the statistical approach.

Additional consideration

There are additional problems, both economic and statistical that are common to nearly all time series analyses. Three are mentioned here: (1) omission of relevant variables, (2) multicollinearity, and (3) autocorrelation in the residuals.

Specification bias as the result of the omission of relevant variables has been mentioned in an earlier section. This, according to Nerlove (50) has been one of the major sources of error in previous research. The omission of prices of alternative outputs, for example, may lead to a negative elasticity of supply. However, the high degree of multicollinearity between economic time series makes it impossible to include all relevant variables. Compromise is achieved by selecting the most relevant variables and allowing these to explain changes in variables not included. Hence, even at best, the time series model normally contains some specification bias. This points to the advantage of regional time series analysis. Regional analysis reduces the number of relevant alternatives. They can, therefore, be more easily incorporated into the model.

Multicollinearity is often the result of correlated time trends. This suggests that the use of first difference or of ratios and deflators will help to reduce this problem. Multicollinearity cannot be eliminated without removing the fluctuations which give economic significance to the series. Price series, in particular, tend to move together under the common influence of economic events.

The regression model assumes no serial or autocorrelation[a] in the residual. Autocorrelation may arise from, (1) faulty choice of the form of the relationship, (2) omission of relevant variables, and (3) errors of observation. Systematic errors may occur in either of the first two instances because most economic time series are positively autocorrelated. Systematic errors often occur in observed series due to the manner in which the data is compiled. A mistake in one

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[a]There appears to be some confusion of these terms in the literature. Foote (25) uses serial correlation and autocorrelation interchangeably. Tintner (57, p. 187), however, states, "By autocorrelation we understand the lag correlation of a given series with itself...By serial correlation we understand the lag correlation between two different time series."

year's observation will often be incorporated into the figures for subsequent years. The presence of autocorrelation in the residuals leads to a loss of statistical efficiency. As the error term becomes more random, the standard errors of the regression coefficient will decline. The Durbin-Watson (20) test for autocorrelation is widely used. Where the test indicates that the residual is serially correlated, Cochrane and Orcutt (13) suggest the use of first differences to randomize the error term.

PRODUCTION AND PRICING PATTERNS

This section contains a description of the production trends, production alternatives, price movements, and pricing policies in the three regions analyzed in the study. The description and graphic analysis represents a preliminary step to more formal statistical analysis. The formulation and examination of tables and graphs assists in the identification of relevant variables and growth trends. It also facilitates the development of *a priori* hypotheses that can be tested through statistical analysis. Such hypotheses serve as a check on the logic of the statistical results. That is, the rejection of a hypothesis will encourage the investigator to search for possible sources of error or bias in the coefficients.

Production Trends

The graphs and tables compare production and prices for the three regions and the United States. Figures 1, 2, and 3 show milk output, production per cow, and cow numbers. In these graphs the annual observations are expressed as a percent of the mean for each of the series for the period 1926-58. This procedure facilitates regional comparison.

Trends in milk production (Figure 1) among regions have had certain similarities. Milk output has increased in all regions. Decline in production in the mid 1930's was followed by an increase during the war years and a second decline in

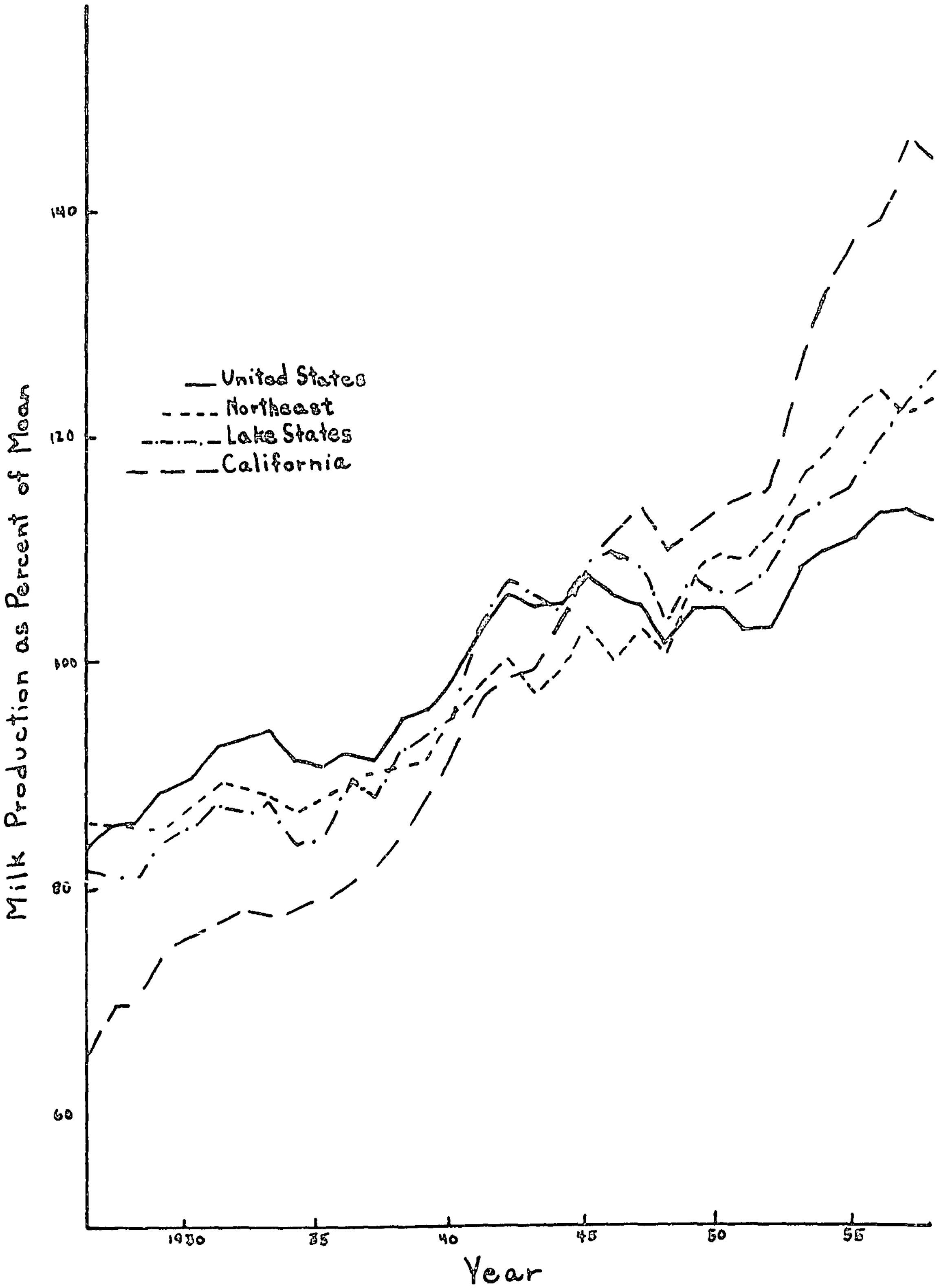

Figure 1. Total milk production on farms in the United States, Northeast, Lake States, and California, 1926-58. Annual observations expressed as percent of mean.

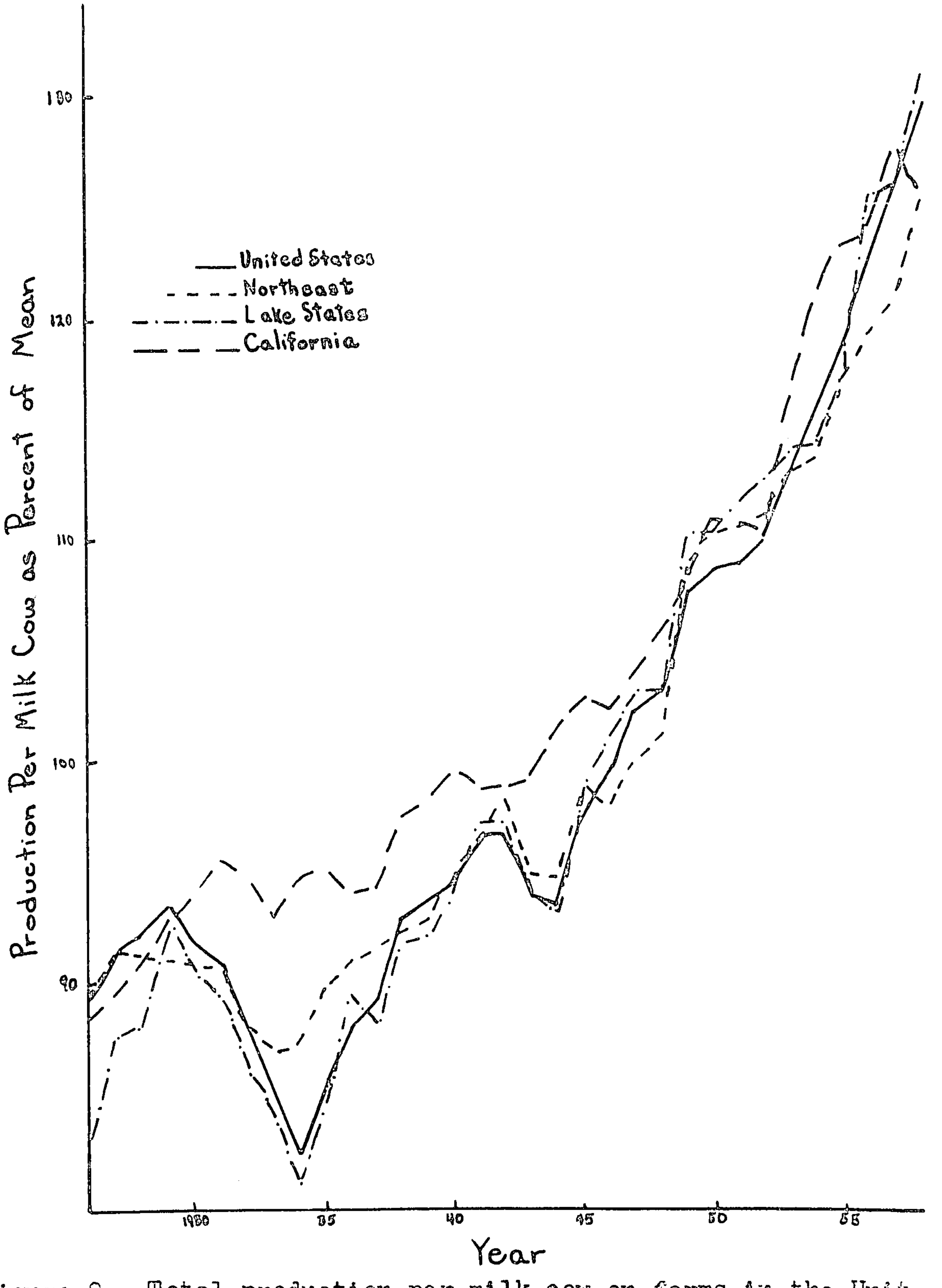

Figure 2. Total production per milk cow on farms in the United States, Northeast, Lake States, and California, 1926-58. Annual observations expressed as percent of mean.

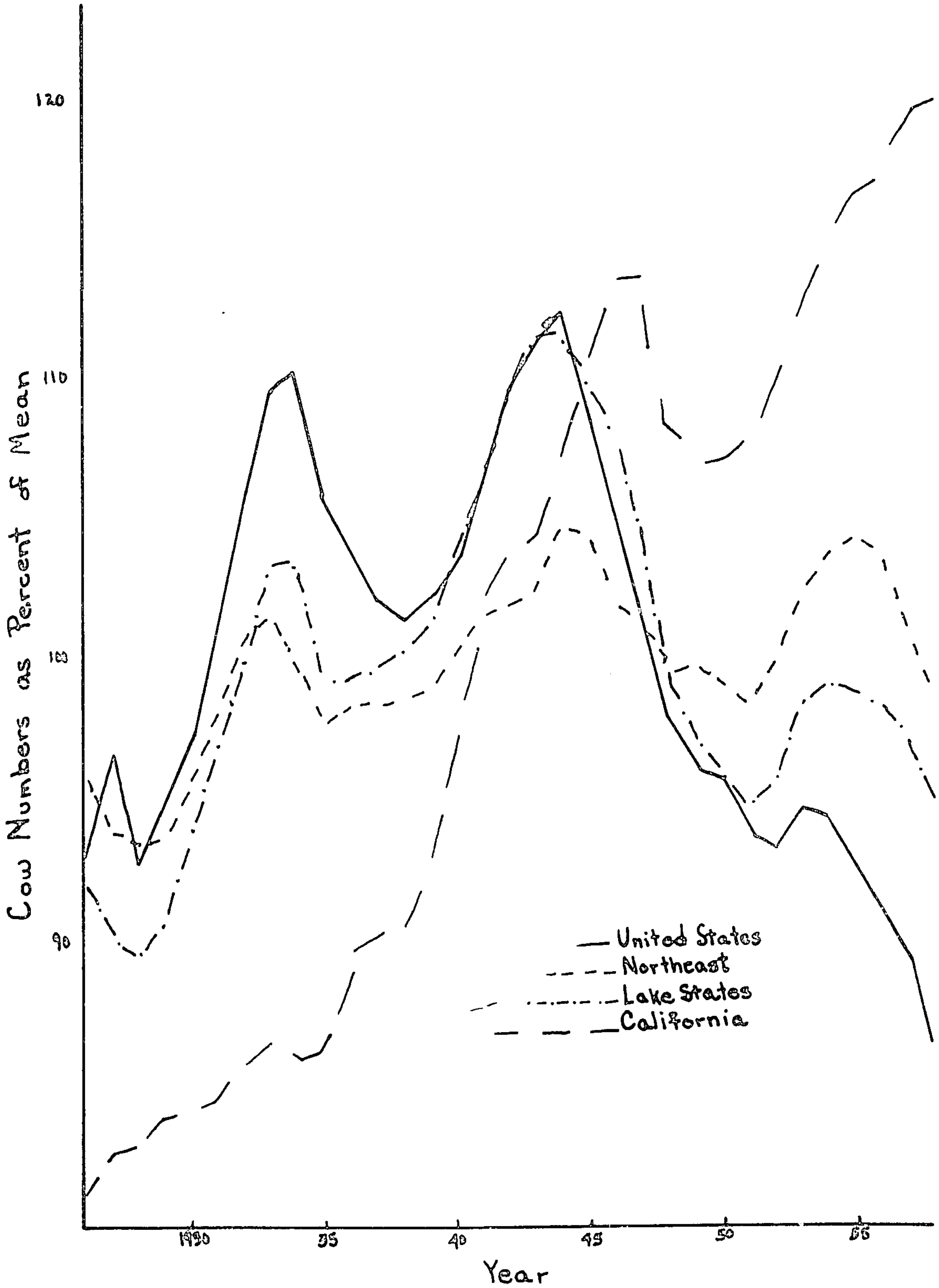

Figure 3. Total number of milk cows on farms in the United States, Northeast, Lake States, and California, 1926-58. Annual observations expressed as percent of mean.

the immediate post-war period. The most recent rise that was initiated in the Korean war shows signs of abating in the Northeast and California.

Table 1. Percent average annual rate of growth in milk output and production per cow for the United States, the Northeast, the Lake States, and California in specified time periods.

	United States	Northeast	Lake States	California
Milk output				
1926-58	1.0	1.1	1.4	2.5
1926-40	1.3	.6	1.4	2.2
1947-58	.5	1.7	1.3	2.6
Production per cow				
1926-58	1.4	1.2	1.2	1.1
1926-40	.1	.1	.1	.8
1947-58	2.2	2.0	2.2	1.8

The rate of growth in production has varied not only between regions, but also between different time periods within regions. Table 1 shows the average annual rate of growth in milk output for the entire thirty-three year period and for the prewar and postwar years. The more rapid postwar growth in the Northeast contrasts with the decline in rate of growth for the United States. Rate of growth in California is almost double that in other regions. However, the rate of growth for all three regions has exceeded that of the United States, which indicates an increasing concentration of production in these three regions. Between 1926 and 1958 the proportion of the nation's production represented by these regions rose from 45 to 52 percent.

Long-run growth in output has been achieved in most regions by an increase in production per cow. Production per cow has been highest in California. Figure 2 shows that California production per cow was not affected seriously by depression or drought in the 1930's. In the postwar years, however, average annual rate of increase in the other two regions and in the total United States has surpassed that for California (see Table 1). The differential for production per cow between regions has declined.

Gains in production per cow are largely a result of the increase in grain fed per cow. There has been some improvement in the quality of ration. However, there is no evidence as in the case of most livestock products, of improved feed conversion efficiency. (Conversion efficiency in milk production is considered to be related to the ability to convert feed into milk after body maintenance requirements are met.)

The similarity of regional trends in production per cow indicates that the milk production differential between regions is altered primarily through changes in cow numbers. In recent years the decline in cow numbers in the total United States has been more rapid than in the specialized dairy regions. This has resulted in the growing concentration of production in these regions. Cow numbers have risen in California to keep pace with the rapidly increasing population. By contrast, in some other sections of the country more profitable alternatives have encouraged the movement of resources out of dairying. The decline of dairy cows in southern Minnesota and Iowa, for example, has been very rapid.

Comparison of Figures 1 and 3 shows that cyclical fluctuation in cow numbers has been related to fluctuation in total milk production. However, variation in production is less pronounced because a change in cow numbers is inversely related to production per cow. For example, a decline in cow numbers has the short-run effect of increasing production per cow since low producers are culled first.

Discussion to this point has focused on trends in production. It is customary to account for long-run secular trend by the use of a variable such as time. This study, is concerned with short-run response to price. Therefore, regional differences in production response which are not secular in nature are examined next.

Short-run Production Response

The short-run production response for the three regions studied and for the United States is compared in the next two tables (Tables 2 and 3). Table 2 shows the average percent year to year change observed in milk production, production per cow, and cow numbers from 1926-58. Year to year change in milk output is lower in the United States than in any of the three regions studied. The strong upward trend in population accounts for the comparatively high rate of change in California. Average year to year change for production per cow and cow numbers is highest in the Lake States and the

United States (values are identical) and lowest in the Northeast.

Table 2. Percent average year to year changes observed in milk production, production per cow, and cow numbers in the United States, the Northeast, the Lake States, and California, 1926-58.

	United States	Northeast	Lake States	California
Milk output	1.8	2.1	2.5	3.3
Production per cow	2.2	1.8	2.2	2.1
Cow numbers	1.9	1.4	1.9	1.8

The variability of deviations from the trend is shown in Table 3. Trend was removed by the following procedure. Observations for milk production and for the milk-feed price ratio were converted to logarithms. Regressions were computed with time as the independent variable, as shown in Equation 10.

$$\log Q_m = \log a + \log b(T) \tag{10}$$

where:

Q_m = the quantity of milk

T = time: 1926 = 1

$\log a$ = a constant

$\log b$ = the coefficient of time

Observations for time were expressed in natural numbers with 1926 = 1, 1927 = 2, etc. except in the Northeast. In this latter region the squared values of time were used (i.e., 1926 = 1, 1927 = 4, etc.). This form of the trend variable gave a better explanation of growth in Northeastern production. (Table 1 shows that in contrast with other regions average annual rate of growth in the Northeast has been much higher in the postwar than in the prewar years.) The standard error of the estimate (which represents the standard error of the deviation from the trend in Equation 8) was divided by the mean for each series. The values obtained for each region were expressed as a percentage of the corresponding value for the United States.

Table 3. Index of variability for milk production and the milk-feed price ratio showing variability about the 1926-58 trend for the regions (the Northeast, the Lake States, and California) as compared with variability similarly measured for the United States.[a]

	United States	Northeast	Lake States	California
Milk output	100	71	125	125
Milk-feed price ratio	100	85	148	110

[a]Computed by dividing the ratio of the standard error of the estimate to the mean for each area by the same ratio for the United States and multiplying by 100.

This table emphasizes the comparative stability of the Northeast both in production response and in price change. Variability in milk production is highest in the Lake States and California. Variability in the milk-feed price ratio is considerably higher in the Lake States than in the other regions. This volatile milk-feed price ratio is due to (1) a more elastic consumer demand for manufactured dairy products, in contrast to fluid milk, and (2) a more elastic farm demand for feed grain because of the wider alternative uses for feed.

The following three subsections describe in further detail the factors which influence short-run production response in each of the regions.

The Lake States

Between 75 and 80 percent of the milk produced in the Lake States (Michigan, Wisconsin, and Minnesota) in the past three decades has been used for manufactured dairy products. Figure 4 shows that the percentage of milk going into manufacturing channels has been very stable except during the war years when the fluid demand rose sharply. In spite of this stability the Lake States region is a heterogeneous producing area. Production changes in neighboring states and in neighboring geographical regions within states have not been similar.

The proportion of Michigan milk for fluid use has risen steadily from 35 percent in 1940 to over 60 percent at present. It appears that urban expansion and industrial growth have slowed the increase in production but have enhanced fluid demand. From the period 1946-58 Michigan cow numbers declined by 22 percent.

Wisconsin is the number one dairy state in the region and in the country in terms of total production. Wisconsin milk production has been about equal to that of the other two states combined. Growth in production has been rapid in this state where farm and non-farm alternatives to dairying have been limited. During the period when Michigan cow numbers were declining rapidly Wisconsin cow numbers fell by only five percent.

Minnesota, like Michigan, has experienced a slow postwar rate of growth in production and a rapid decline in cow numbers (20 percent from 1946-58), but for different reasons. Minnesota's price was adversely affected by the decline in the demand for butter. Farmers responded by transferring resources to more profitable alternatives.

Regional aggregation tends to obscure many of these local differences. However, the most important fact affecting regional response is the concentration of manufactured production. Fewer fixed resources are committed to dairying in manufacturing than in fluid regions. When dairy prices are unfavorable, beef cattle and hog enterprises can be expanded without excessive investment. Barns which house dairy cows can be used for other livestock.

Manufactured milk prices are volatile. Demand for manufactured dairy products is more elastic than demand for fluid milk. Administered prices have not been adopted for manufactured milk. Furthermore, efforts to stabilize production and price in the fluid regions may lead to greater price variability in the manufactured milk regions.

A final factor which has had an important influence on Lake States production is the weather. During the drought

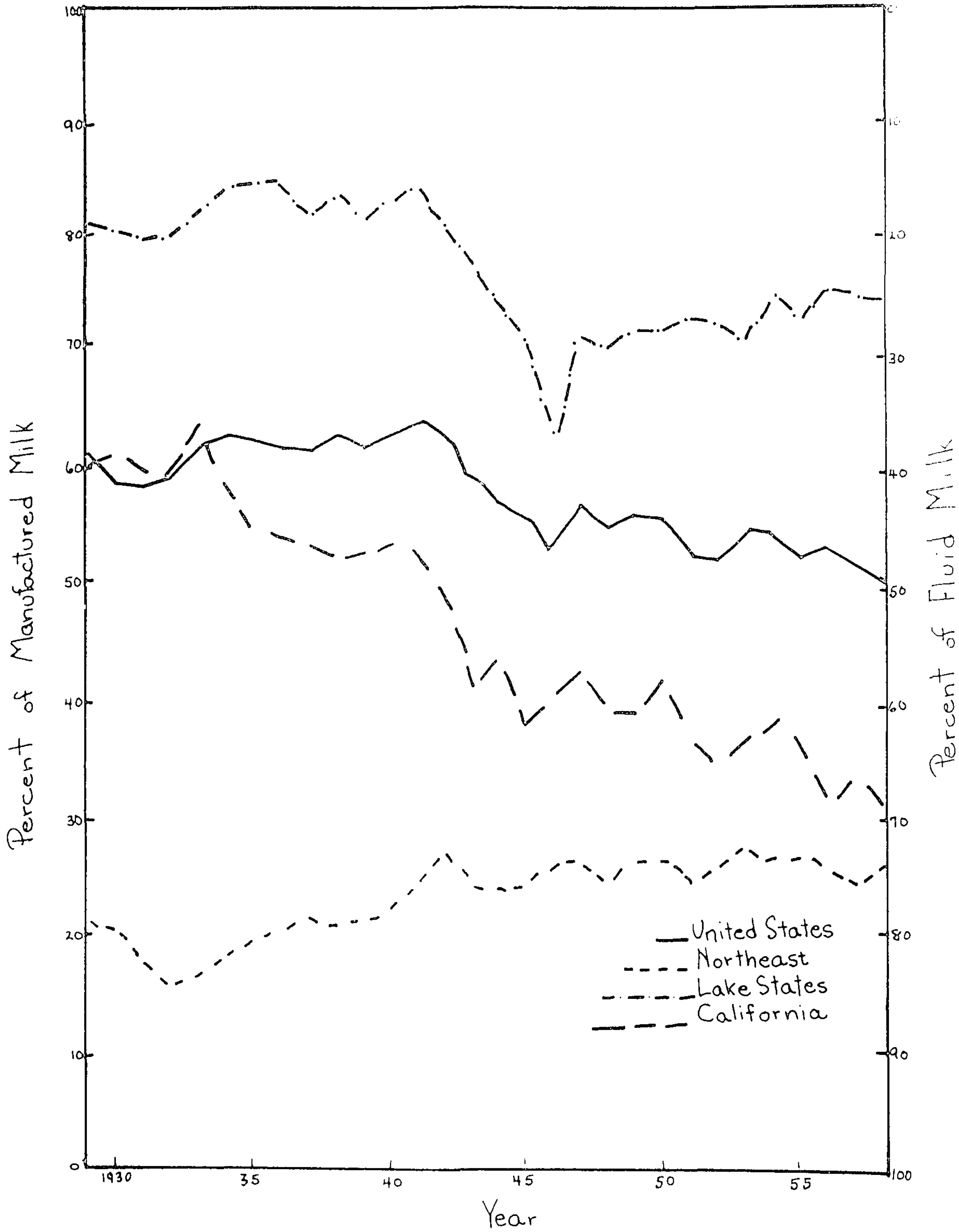

Figure 4. Percent of milk for manufactured and fluid use in the United States, Northeast, Lake States, and California, 1929-58.

years in the 1930's, for example, the decline in grain and forage yields and in production per cow was greater in the Lake States than in the Northeast or California. A portion of the decline in production per cow was due, of course, to economic factors.

<u>The Northeast</u>

Northeast output is consumed principally as fluid milk and cream. As Figure 4 indicates, approximately 75 percent of the production has been sold for fluid milk since the war. This marks a slight decrease from the prewar years.

The Northeast can be divided into two regions based upon growth in milk production: (1) New England excluding Vermont, and (2) the Middle Atlantic States including Vermont, Maryland, and Deleware. There has been almost no upward trend in New England production. Increases in production per cow have been offset by declining cow numbers as more land has been retired from farming. Profitable on-farm alternatives to dairying exist only in localized areas. The low physical productivity of the soil coupled with favorable off-farm opportunities have encouraged the movement of land and labor resources out of agriculture.

Production in the Middle Atlantic States on the other hand has been steadily increasing. This increase in production has been the result of a growing comparative advantage for dairying in areas which hitherto enjoyed a wide range of profitable alternatives. The New York milkshed, for example, has expanded into upstate New York and into sections of Pennsylvania, Maryland, and Deleware bringing higher prices to producers. Milk price has risen relative to prices of many of the important alternatives, cash grain, fruit and vegetables, and particularly poultry. The small poultry enterprises which existed on many dairy farms have almost completely disappeared in the past decade. The economies to scale brought about by technological innovation in both dairying and poultry production have increased the advantages of specialization.

Since a large majority of the Northeastern dairy farmers are fluid producers, more fixed resources are committed to milk production than in the Lake States. This has contributed to the stability of production in the region. Transfer of resources between alternative enterprises is less rapid and less easily accomplished.

Administered pricing has also contributed to the stability of production. The major Northeastern markets adopted federal milk marketing orders in the late 1930's and the early 1940's. Classified pricing of milk was used principally for Class I (fluid) milk. Price formulas in the Northeast have been based upon economic indices reflecting changes in the cost of living. Not all producers are included in the federal order markets. However, the volume of milk entering these markets is sufficient to make the effect of administered pricing felt throughout the entire region. Halvorson (30, p. 1110) suggests that the introduction of administered pricing should increase the certainty of expectation and hence the price elasticity for milk.

It has been shown in Tables 2 and 3 that both milk production and price have been more stable in the Northeast than in either the Lake States or California. A larger portion of the variance in Northeast production is explained by trend. From a statistical point of view it should be more difficult to identify elasticities of response.

California

The pattern of milk production in California represents a sharp contrast to that for the other two regions. Throughout the state there is a wide variety of production conditions and a wide range of production alternatives. The percentage of milk for fluid use has increased from 40 to 70 percent in the period since 1926 (see Figure 4). This increase reflects the rapid growth in population.

Over one-third of California's production is in the San Joaquin Valley. Here, as in other parts of the state, much of the crop and pastureland is irrigated. Irrigation results in higher quality forage and greater stability in forage production. This stability is illustrated by a comparison of coefficients of variation for hay yield per acre. In the 1926-58 period the coefficient of variation was 19.7 percent in the Lake States, 14.6 percent in the Northeast, and only 12.6 percent in California. Of equal importance to dairy production is the wide variety of alternative uses for irrigated land.

Approximately 18 percent of the state's milk is produced in the dry lots of Los Angeles County. These dry lots are not operated as typical farms. All feed and all replacement cows are imported. The management and labor functions are frequently separated. The "farm" is run as a formal business with complete accounting records kept of all transactions. There are no non-farm alternatives to dairying. However, these operations are very sensitive to changes in price. For example, with an increase in the price of beef relative to milk, more cows are slaughtered and more replacements are imported.

California, like the Northeast, has had administered pricing of milk for a number of years. California price regulation was initiated through the Young Act in 1935. The Bureau of Milk Control has the authority to establish minimum prices for fluid milk and cream. Manufactured milk prices, and the difference in cost of production between manufactured and fluid milk are considered in establishing these prices.

In summary, short-run fluctuations in total milk production are comparatively small. Average year to year change in milk production (1926-58) is highest in California, but is only 3.3 percent. Average year to year change is less than two percent for the United States.

Economic factors which influence short-run production response include: (1) the price of milk, (2) the cost of inputs, and (3) the prices of competing products. The major input in milk production is feed which accounts for more than 50 percent of the cost. On-farm alternatives are more important in the Lake States than in the Northeast. In the latter region, however, off-farm opportunities have had a significant effect on the labor force. Alternatives to dairying are important in California, but are too numerous to be identified and incorporated in a regression model for the state.

The amount of capital committed to dairying both on the farm and for the region as a whole has a considerable effect on the response. More fixed capital is required for fluid production and farmers in these regions react more slowly to a change in price. Total farm resources committed to dairying are higher in the Northeast than in the other regions, which sets a limit to the increase in production following a short-run price rise.

A non-economic factor which has influenced milk production is the weather. Rainfall (or lack of rainfall) has had more influence on crop yields in the Lake States than in the other regions.

A Priori Hypotheses

It is possible to form tentative hypotheses concerning the elasticity of response based upon the descriptive analysis of this section. These hypotheses are tested through a comparison of the results obtained from regression analysis.

The three regions analyzed in this study represent the major milk producing areas in the United States. However, over 40 percent of the milk is produced in "non-dairy" areas (i.e., the regions not covered by this study). Dairying in these regions faces strong competition from other farm alternatives. Response to a change in price in many of these areas should be more elastic than in the traditional dairy areas. It is therefore hypothesized that the elasticity for milk in the United States will exceed that for any of the three regions studied.

Since less fixed capital is required in the production of manufactured milk, Lake States resources can be more easily transferred to alternative livestock production than those in the fluid milk sheds. Competition for feed often occurs between two livestock enterprises on the same farm. Of the three regions studied, the elasticity of supply should be highest for the Lake States.

Although the Northeast and California produce principally fluid milk, several factors suggest that California elasticity will exceed that for the Northeast. Much of the land in the Northeast is suited only for hay or pasture. In contrast, there is a wide variety of profitable alternatives for irrigated land in California. In the dry lot operations of California, where there are no alternatives to dairying, operators appear to have a greater economic awareness and to be more sensitive to price change.

Finally, it is hypothesized that the elasticity of supply has risen in the postwar period in the fluid milk sheds (the Northeast and California). One object of the milk marketing orders introduced in the late 1930's was to stabilize prices. Prices have been administered principally for Class I (fluid) milk. Reducing the uncertainty of price change should lead to a more rapid response. Greater technological efficiency and extended knowledge may also contribute to a more elastic response in the postwar period.

EMPIRICAL MODELS

The descriptive analysis of the previous section indicates that no single empirical model will provide the needed information on short-run supply response for milk. The relevant variables and the way in which these variables enter the equation will differ from region to region. Even within a given region present knowledge of farmer behavior is not sufficient to specify the appropriate form of the model. For example, do Northeastern farmers respond to changes in the price of milk alone, or to changes in the price of other alternatives? To these questions of appropriate model specification must be added the problems of reversibility, lagged response, and single vs. simultaneous equations discussed previously.

In order to meet the objectives of this study, it is necessary to consider several models of supply response. Different models (like different techniques) can provide complementary information. For example, comparison of results may reveal inconsistencies. Incompatible answers are often due to problems that are either statistical or economic in origin. The model can often be modified to overcome these difficulties and provide more reliable estimates.

Several models of short-run supply response for milk are set forth in the following pages. With the exception of the simultaneous equation system, the models described have been used in the empirical analysis of this study.

The Simultaneous Equation Model

Equation 11 expresses the identity: the quantity of milk produced is determined by the number of cows and the production per cow:

(11) $$Q_{mt} = (P/C_t)\,(C_t)$$

where:

Q_{mt} = the quantity of milk produced in time period t

P/C_t = the production per cow in time period t

C_t = the number of cows in time period t

It is possible to estimate the quantity of milk directly or to estimate separately production per cow and cow numbers. Consider the first of these alternatives:

(12) $$Q_{mt} = f(P_1, X_1, X_2 \ldots X_n)$$

where:

P_1 = the price of milk

$X_1 \ldots X_n$ = other supply shifters and structural variables which explain changes in milk production

Assume that the quantity of milk produced is a function of the price of milk, and other variables including those describing shifts in supply and structural change (as in Equation 12). The variables on the right hand side of this equation may be either endogenous (i.e., correlated with the residual) or exogenous. If one or more of the variables is endogenous, a simultaneous equation model should be used.

In dairying, response to a change in price can be almost instantaneous (e.g., farmers can quickly change the feeding or culling rate). However, past investigations indicate that there is at least a three month's lag between the price change and the production response. The lagged response suggests a least squares approach. However, when annual data are used this initial production response will occur in the same time period as the price change. Therefore, it would be possible to consider the response in the simultaneous framework such as the one outlined below:

P_m, Q_m; Y	(Demand for milk)
P_m, Q_m, P_{ff}; P_l	(Supply of milk)
P_m, P_{ff}, Q_{ff}; P_l	(Demand for feed)
P_{ff}, Q_{ff}; Q_{fp}, P_{fl}, P_{fs}	(Quantity of feed fed)
P_{fp}; $P_{ff(t-1)}$, $P_{fl(t-1)}$, $P_{fs(t-1)}$, R	(Quantity of feed produced)

where:

P_m = price of milk

Q_m = quantity of milk produced

Y = per capita consumer income

P_{ff} = price of feed fed to dairy cows

P_l = price received for competing livestock (hogs and beef)

Q_{ff} = quantity of feed fed to dairy cows

Q_{fp} = quantity of feed produced

P_{fl} = price of feed fed to livestock other than dairy animals

P_{fs} = price of feed for storage

R = rainfall during the cropping season

The variables to the right of the semi-colon are exogenous, those to the left are endogenous. The quantity of feed produced is exogenously determined. The demand and supply relations for milk and for feed fed form a simultaneous system. It would of course, be possible to expand this model to include demand and supply equations for several livestock products. Feed could also be classified in several categories (e.g., concentrates, grain, forage).

Simultaneous models of this general nature require an accurate description of the farm decision-making framework in terms of the variables selected. A priori knowledge is often insufficient to indicate with any degree of certainty even the sign of the coefficient. Causes of the cattle cycles, for example, are still frequently disputed. The influence of government storage and support programs on the demand and supply patterns for feed is not well understood. It is difficult to obtain adequate data on forage production and quality. The unsatisfactory results obtained by Hildreth and Jarrett (34) and Dean (17) emphasize the need for more accurate information if simultaneous equation models are to prove useful in supply analysis.

Single Equation Models

Single equations have been used primarily in this study. It has been shown that the use of single equations where simultaneous relations exist may lead to biased results. However, the majority of the variables which affect milk production are predetermined (i.e., the response is lagged at least one year). This bias, therefore, may not be serious. In addition, Christ (14) has pointed out that the limited information procedure does not guarantee results which are statistically more reliable. Hildreth and Jarrett (34) in their model of the livestock economy, compared coefficients obtained by these alternative methods and did not find large differences.

There are distinct advantages to be gained through the use of least squares. First, the ease of computation permits more attention to be directed to the consideration of the variables and the form in which they should be introduced. This seems advisable in view of the limited knowledge concerning farmer response. In contrast, the lengthy computations for simultaneous systems discourage modifications and corrections even when it becomes apparent these should be made. Secondly, least squares can be more readily combined with graphic analysis. Graphs and scatter diagrams may serve as a useful guide in model construction.

Time series analysis could appropriately be considered a series of steps beginning with graphic analysis and ending with the more sophisticated simultaneous equation models. Each step provides information which will make it possible to move one step further (e.g., a graph may establish the shape of a trend for a least squares model). Thus single and simultaneous equation analyses are complementary rather than mutually exclusive.

Three basic forms of the single equation have been employed: (1) the traditional model, (2) the distributed lags model, (3) the irreversible model. In all three instances milk output is considered as the dependent variable. Independent variables have been introduced with observations of variables expressed as ratios (e.g., price of milk divided by the price of feed). Variables have also been modified through other methods where this appeared appropriate. For example, moving averages have been used to remove short-run fluctuations. The variables are defined and the form of the variables explained in detail in each of the sections containing the empirical results.

Computations have been made principally for logarithms of observed values of variables and for first differences of logarithms. In terms of natural numbers the regressions are curvilinear. This assumes a multiplicative relationship between independent variables as shown in Equation 13.

$$Q_m = a(P_m/P_f)^{b_1} (P_m/P_b)^{b_2} \tag{13}$$

where:

Q_m = the quantity of milk produced

P_m/P_f = the ratio of the price of milk to the price of feed

P_m/P_b = the ratio of the price of milk to the price of beef

a = a constant

b_i = the elasticity for the ith variable

The values b_1 and b_2 are the coefficients of the variables expressed as logarithms in the regression equation.

The identity in Equation 11 indicates the logic of the multiplicative assumption for milk production. In Equation 13 the variable which accounts for change in production per cow, (P_m/P_f), is multiplied by a variable which explains change in cow numbers, (P_m/P_b). Where multiplicative relationships hold, Foote (25, p. 37) indicates that: (1) relations will be more stable in the percentage than in absolute terms, and (2) the unexplained residuals expressed as percentages will be more uniform over the range of the independent variables. (When the variance is uniform the residuals are said to be homoscedastic.)

The first difference of logarithms (transformed to natural numbers) expresses the observation this year as a percentage of the observation in the previous year. First difference analysis focuses on short-run year to year changes. At the same time, it serves as a check on the logarithmic models where serial correlation of the residuals or multicollinearity may lead to questionable results. If the residual of the logarithmic equation is autocorrelated, transformation to first differences will often reduce the errors of the coefficients. The confidence interval of the estimate is thus narrowed. Multicollinearity is frequently the result of correlated trends in independent variables. Therefore, removing the effect of trend by first differences may reduce this problem.

The trend variable in most regressions has not been converted to logarithms. Time was expressed as an exponential function. The exponential is illustrated in Equations 14 and 14A.

(14) $$Y = aX^b c^t$$

(14A) $$\log Y = \log a + b \log X + (\log c)t$$

where:

Y = dependent variable

X = independent variable

t = time

a = a constant

b = coefficient for the independent variable X

c = a constant coefficient for the trend variable t

In most of the equations in this study $t = 1$ in 1926, 2 in 1927 etc.

The exponential function was used in preference to the power function (i.e., time converted to logarithms) because the exponential better describes the trend in milk output and production per cow (see Figures 1 and 2). In all cases regional production has increased at an increasing rate over the period of this analysis. The rate of increase in the regression model is determined by the value of the constant c (in Equation 14). This value is normally close to but slightly greater than one. This constant is raised to a successively higher power in each time period. In the power function, on the other hand, increasing values of t are raised by a constant amount c. The value of the exponent remains unchanged and is normally close to zero. This means that trend increases at a decreasing rate. The use of the power function to explain production growth in the regions studied would bias the results.

Distributed lags

The response to a change in price may be lagged over several years. To obtain a short-run estimate of supply it is not necessary to approximate the complete time path of adjustment. Therefore, emphasis has been placed upon an intermediate model for distributed lags. The form of the intermediate model is shown in Equation 15.

$$(15) \quad Y_t = a + b_1P_t + b_2P_{t-1} + b_3X_1 + b_4X_2 \ldots b_{n-2}X_n + u_t$$

where:

Y = output of milk in time period t

P_t = the price of milk in time period t

$X_1 \ldots X_n$ = variables other than the price of milk in the regression equation

a = a constant

b_i = the coefficient of the ith variable

u_t = a random error

In this model the short-run elasticity of supply is the sum of the coefficients b_1 and b_2 (assuming computation in logarithms).

The Nerlove model has been employed to determine the magnitude of long-run as well as short-run supply response. The difficulties likely to be encountered in the use of this model have been discussed previously. The general form of the model used is shown in Equation 16.

$$(16) \qquad Y_t = a + b\gamma P_{t-1} + (1-\gamma) Y_{t-1} + u_t$$

where:

Y_t = quantity of milk produced in time period t

P_{t-1} = the price of milk in the previous year

b = the coefficient of long-run reaction

γ = the coefficient of adjustment

u_t = a random error

The model assumes a coefficient of expectation, $\beta = 1$.

Irreversibility

The hypothesis that the elasticity under rising price exceeds the elasticity under falling price is normally associated with the mobility of fixed assets. The amount of fixed capital required in dairy farming is comparatively high. Short-run changes in production per cow can occur without a change in fixed assets. Milk production can be varied by altering the feed ration or the culling rate. These short run changes must be distinguished from longer run changes necessitating an expansion or contraction of livestock facilities and other fixed assets. Expansion of facilities in response to a price rise is assumed to occur more rapidly than contraction under a price fall of equal magnitude. On the other hand, short-run supply curves are assumed to be more nearly reversible.

Halvorson (30) used the Nerlove model to distinguish between short and long-run supply response. He found the elasticity of supply under falling prices to be higher than under rising prices in both adjustment periods. A different method of considering both long and short-run price movements has been used in this study. The procedure is described as follows.

First, observations were obtained for the milk-feed price ratio. Then, the years 1926-58 were divided into periods of long-run rising and falling prices. The deflated milk price was examined to assist in drawing the lines between periods. These intervals of long-run price movement were advanced two years to make allowance for a lag in the adjustment of cow numbers. Within periods the price change was determined for each year. There were thus four categories of observations for long and short-run milk price changes: (1) rising long-run, rising annual price, (2) rising long-run, falling annual price, (3) falling long-run, falling annual price, (4) falling long-run, rising annual price. Finally, regressions were computed for each of these groups. The variables used in these models were time and the prices of milk and feed. Short-run elasticities were thus obtained under four combinations of long and short-run price change.

The Recursive System

Since milk production can be varied through changes in production per cow or cow numbers, it is important to consider the relation of these components to the change in total production. In previous research estimates of elasticities have been determined independently for these two components. Halvorson (31) pointed out, however, that production per cow is related to the number of cows. He reasoned that a decline in cow numbers will cause production per cow to rise because the cows eliminated will be the poorest producers. Conversely, rapid expansion in herds must occur through (1) reduced culling rates or (2) transfer of dual purpose cows to dairy production. This relationship provides the basis for the recursive or sequential model.

The sequential pattern is described as follows. The number of milk cows in the current year is determined by events which occurred prior to this year. The independent variables in the equation for cow numbers are lagged at least one time period. These predetermined variables explain changes in cow numbers. Cow numbers in the current year is in turn one of the variables used to explain changes in production per cow. The inverse relationship between production per cow and cow numbers is simultaneous. (For example, a drop in cow numbers means an immediate increase in production per cow.) Therefore, variables representing production per cow and cow numbers are both endogenous (i.e., correlated with the residual). The equation with cow numbers as the dependent variable can be computed first. The expected value for cow numbers can then be substituted for the observed value in the equation for production per cow. This substitution of the expected value removes the correlation between cow numbers and the residual. Cow numbers thus becomes an independent variable in the equation.

Given the identity, milk production equals production per cow multiplied by cow numbers, the milk price coefficients from these two equations can be combined to obtain an elasticity of response for total milk production. The steps in obtaining this elasticity are shown in Equations 17 through 21. The equations are hypothetical but show the basic relationships.

(17) $$C_t = a(P_{m(t-1)})^{b_1} (P_{b(t-1)})^{b_2}$$

(17A) $$P/C_t = c(P_m/P_f)_t^{d_1} (C_t)^{d_2}$$

where:

P/C_t = production per cow in the current year

C_t = dairy cow numbers in the current year

P_m/P_f = price of milk divided by price of feed

$P_{m(t-1)}$ = price of milk in the previous year

$P_{b(t-1)}$ = price of beef in the previous year

a and c = constants

b_1 and d_1 = elasticities of variables

Since cow numbers are predetermined, Equation 17 is solved first by least squares and the values for C (estimated number of cows) substituted into Equation 17A. The substitution of the expected value of C is necessary because the observed values of C in Equation 17A are endogenous or correlated with the residual. Equation 17A is then computed using least squares. The results can be substituted into the identity (Equation 18) to determine the elasticity of supply for milk (where Q_{mt} equals the quantity of milk in the current year). This substitution is shown in Equation 19.

$$(18)\ Q_{mt} = (C_t)(P/C)_t$$

$$(19)\ Q_{mt} = \left[a(P_{m(t-1)})^{b_1}(P_{b(t-1)})^{b_2}\right]\left[c(P_m/P_f)_t^{d_1}(C_t)^{d_2}\right]$$

In order to complete the solution it is necessary to substitute the righthand side of Equation 17 for C in Equation 20.

$$(20)\ Q_{mt} = \left[a(P_{m(t-1)})^{b_1}(P_{b(t-1)})^{b_2}\right] \left[c(P_m/P_f)^{d_1}(a\left\{P_{m(t-1)}\right\}^{b_1}\left\{P_{b(t-1)}\right\}^{b_2})^{d_2}\right]$$

Collecting terms we have:

$$(21)\ Q_{mt} = \left[a(P_{m(t-1)})^{b_1}(P_{b(t-1)})^{b_2}\right]^{d_2+1} \left[c(P_m/P_f)^{d_1}\right]$$

The elasticity for milk production, <u>ceteris paribus</u>, is represented by the sum of the coefficients d_1 and $b_1(d_2+1)$. The coefficient d_1 is the exponent of the milk-feed price ratio in Equation 21. The value $b_1(d_2+1)$ is the exponent of the milk price lagged one year. The value d_2, which is the coefficient for cow numbers in Equation 17A, should be negative since production per cow and cow numbers are inversely related. The total effect of a change in cow numbers on milk production in the short-run must take into account this inverse relationship.

The recursive model outlined above appears to be a logical theoretical formulation of milk supply response. In addition to estimates of response to total milk production, the sequential analysis provides information on the relative importance of response through changes in cow numbers and production per cow.

SUPPLY RESPONSE IN THE LAKE STATES

The various forms of the empirical models employed in this study were discussed in the previous section. Results from regression analysis are presented in this and the following sections in equation and tabular form. Regressions have been computed to show the effect on milk price elasticities of: (1) the addition of variables to the model, (2) the introduction of distributed lags, (3) the grouping of years according to time periods, and (4) the grouping of years based upon rising and falling milk prices. In developing the recursive model, equations are also presented using production per cow and cow numbers alternatively as the dependent variables.

Except for the irreversible model, regressions for the Lake States have been computed for two time periods, 1926-58 and 1947-58. The comparison of elasticities should indicate whether the rapid postwar technological advance has affected the elasticity of response. The principal variables used other than trend were the prices of milk, feed, and competing livestock products. From the previous description of the area these were thought to be most relevant. Profitable alternative uses for feed grain are more numerous in the Lake States than in the other regions studied.

Regression equations presented are principally for observations in logarithms. However, a limited number of equations have been estimated with observations in first differences of logarithms. As previously noted, transformation of observations to logarithms implies a multiplicative relationship among the independent variables. The coefficients of the variables represent elasticities with respect to milk supply.

A brief discussion of the shortcomings of the data used is presented in Appendix A. The major time series for each region are presented in separate tables in this same appendix. Time series for the Lake States are in Tables 39 and 40. Each column heading indicates the units in which the variables are computed. Table 47 in Appendix A consists of bibliographical references showing the published sources from which the data were obtained.

Conventional Single Equations

Tables 4 through 7 include the coefficients estimated by regressions with milk production as the dependent variable. In these tables the number of the equation and the time period are indicated in the lefthand column. The name and numerical designation of the variables are shown at the top of each column, e.g., $L5_t$. (The abbreviations used in the heading are defined in the footnote at the bottom of the table.) The letter L in the numerical designation refers to Lake States; the number 5 indicates the number of the variable in the list of definitions. These definitions appear in the text in conjunction with each table. The letter t refers to the time period. The principle followed in this text is to consider the year t the period in which the dependent variable is measured. Time period t refers to the "current year." Time period t-1 refers to the "preceding year." The designation t-1 following the number of the variable thus indicates that the observations have been lagged one year.

The coefficients of the variables for each equation are listed in the column below the heading. Directly beneath each regression coefficient in parantheses is the standard error. The statistical t value (not to be confused with time period t) may be obtained by dividing the regression coefficient by its standard error. The value can be checked against tabulated values of the t distribution to determine whether or not the coefficient is statistically significant.

Since the sign of the coefficient is normally dictated by logic, it is customary to use a one-tailed t test. The t value at the five percent level of significance will depend upon the degrees of freedom in the equation. The number of degrees of freedom is in turn related to the number of observations and the number of variables. (The degrees of freedom equals the number of observations minus the number of variables minus one.) In general, however, t values which are much below two will not be significant at the five percent level.

The first column to the right of the coefficient of variation contains the R^2, or coefficient of linear determination for each equation. The R^2 indicates how much of the variance in milk production has been explained by the variables included in the equation.

Figures in the next column show the sum of the elasticities for price based upon the assumption that all other things remain constant. (For example, technology and the prices of feed and beef remain unchanged.) This sum represents the absolute value of coefficients of variables containing the price of milk (e.g., milk-feed and beef-milk price ratios). When milk price appears in the denominator the sign of the coefficient should logically be negative, indicating an increase in production with an increase in milk price. In summing the elasticities, the value of coefficients whose signs were logically incorrect were omitted. These values with "wrong" signs were not significant and contributed little to the explanation of milk supply.

The Durbin-Watson d' statistic appears in the column at the extreme right in Tables 4, 7, and 8. The statistic may be used in conjunction with the tables developed by Durbin and Watson (20), to test for serial correlation in the residuals. The presence of serial correlation results in a loss of statistical efficiency.

The Durbin-Watson statistic has been computed for the equation in the 1926-58 period with the lowest error of the estimate. This equation is not necessarily the one which contains the most variables. The addition of a variable must increase the R^2. However, the increase in the error of the estimate due to the loss of one degree of freedom may exceed the decrease in the error of the estimate due to the addition of a variable.

The d' statistic is obtained by the following formula:

(22)
$$d' = \frac{\sum_{t=2}^{N} (d_t - d_{t-1})^2}{\sum_{t=1}^{N} d_t^2}$$

where:

d' = the Durbin-Warson statistic

d_t = the residual in time period t

The d' value is thus the sum of squares of the first differences of the residuals divided by the sum of squares of the residuals. This value d' ranges from zero to four.

Durbin and Watson (20) have developed statistical tables for a two-tailed t test at the five percent probability level. The approximate upper and lower bounds in the Durbin-Watson tables depend on the number of observations and the number of variables in the equation. Comparison of the values for d' and (4-d') with the upper and lower bounds may indicate either the presence or absence of serial correlation, or that the test is inconclusive.

In general, it can be said that: (1) a d' value which is small (close to zero) indicates positive serial correlation, (2) a value which is large (close to four) indicates negative serial correlation, and (3) a value which is close to two indicates no serial correlation. However, as the number of observations decreases the chance of obtaining an inconclusive test increases. Since tables have not been developed for fewer than 15 observations the Durbin-Watson statistic is computed only for the 1926-58 equations.

The variables used in the equations shown in Table 4 are defined as follows:

L_{1t} = the total milk production in the Lake States in the current year. This is the dependent variable.

L_{4t} = time: 1926 = 1, 1927 = 2, 1928 = 3...

L_{5t} = milk-feed price ratio in the current year. The price per cwt. of combined milk and cream marketing in the milk from the Lake States is divided by price per cwt. of concentrate rations fed to milk cows (average for East North Central and West North Central).

L_{5t-1} = milk-feed price ratio in the previous year

L_{5t-2} = milk-feed price ratio two years previous

L_{6t-1} = beef-milk price ratio in the previous year. The price per cwt. canner and cutter grade cows, Chicago is divided by the price per cwt. of milk from combined milk and cream marketing in the Lake States.

$L_{7't-2}$ = hog-milk price ratio two years previous. A variable moving average of price per cwt. packer and shipper purchases in Chicago is divided by the price per cwt. of milk from combined milk and cream marketing in the Lake States.

The milk price in the year t for the Lake States represents an average of the price per hundredweight combined milk and cream marketings weighted according to the volume of milk sold in each state. Feed grain prices by states were not available for the entire period 1926-58. The regional series used, an average of the East North Central and West North Central, was found to be highly correlated with prices for the years state data were available. Observations of hog and beef

Table 4. Milk supply response, Lake States, 1926-58 and 1947-58: showing regression coefficients, standard errors, R^2, the sum of the elasticities for milk, and the Durbin-Watson statistic for regression equations using logarithms of observed values.[a]

Equation Number	Time L_{4t}	Milk-feed p. ratio L_{5t}	Milk-feed p. ratio $L_{5(t-1)}$	Milk-feed p. ratio $L_{5(t-2)}$	Beef-milk p. ratio $L_{6(t-1)}$	Hog-milk p. ratio $L_{7'(t-2)}$	R^2	Sum of E for milk price	D-W stat. d'
(23)	.005615	.24619					.966	.246	
1926-58	(.0001989)	(.04198)							
(23A)	.006815		.23606				.955	.236	
1947-58	(.0005242)		(.05199)						
(24)	.005602	.20078	.11257				.973	.313	
1926-58	(.0001812)	(.04180)	(.04199)						
(24A)	.006621	.03666	.23581				.955	.273	
1947-58	(.0007842)	(.09666)	(.06133)						
(25)	.005596	.20228	.10043	.02821			.973	.331	
1926-58	(.0001836)	(.04237)	(.04622)	(.04227)					
(25A)	.006763	.03558	.22312	.02873			.957	.287	
1947-58	(.000859)	(.10019)	(.06188)	(.05611)					
(26)	.005653	.20533	.10050	.02602	-.01050		.973	.342	
1926-58	(.0002232)	(.04337)	(.04675)	(.04303)	(.02267)				
(26A)	.005489	.09448	.20309	-.04678	-.06328		.992	.361	
1947-58	(.0004866)	(.04975)	(.03012)	(.03100)	(.01268)				
(27)	.005632	.22293	.10755	.01462	-.01210	.05757	.976	.357	1.159
1926-58	(.0001906)	(.03816)	(.04006)	(.03719)	(.01935)	(.03069)			
(27A)	.005467	.09676	.20370	-.04671	-.06293	-.004726	.992	.368	
1947-58	(.000502)	(.05135)	(.03025)	(.03094)	(.01281)	(.02723)			

[a]Abbreviations for column headings are as follows: p. = price, E = elasticity, D-W = Durbin-Watson, stat. = statistic.

prices for the current year are for the Chicago market.

The logic of the form of the trend variable has been discussed in the previous section. Values for time were not converted to logarithms as the exponential function was found to give a better explanation of the milk production trend in the Lake States. In all equations close to 90 percent of the variability was due to trend alone.

Observations of variables other than trend were prices of milk, feed, hogs, and beef. It was thus assumed in the equations in Table 4 that production response represented by deviations from the trend could be explained by economic factors. Observations of variables were used as ratios (e.g., the milk-feed price ratio). This procedure implicitly removes the influence of variations in the general price level and at the same time conserves one degree of freedom. The two prices that form the ratio (i.e., the numerator and the denominator) are assumed to have nearly equal effect on milk supply.

The milk-feed price ratio is commonly used in studies of milk supply response. Cost of feed grain fed to dairy cows is one of the largest items of expense. However, although feed represents a cost to the dairy farmer, a change in the prices of feed may reflect a change in the price of competing livestock products. For example, a rise in the price of beef will enhance the demand for feed. Therefore, the decline in milk production in response to a feed price rise may be due to (1) an increase in the cost of feed, and (2) an increase in the profits from alternative livestock enterprises, or (3) a combination of these two.

The independent variables in Equation 23 and 23A are trend and the milk-feed price ratio. The milk-feed price ratio for the previous year was used in the latter equation. Empirical estimations of subsequent equations (Equations 24A and 25A) indicated that the major response in the 1947-58 period was lagged one year. The regression coefficients or elasticities for the milk-feed price ratio in both time periods are statistically significant and close to .24. It is predicted that a one percent increase in the milk-feed price ratio will bring about a .24 percent increase in milk production.

A third variable has been added in Equation 24 and 24A so that both equations contain the milk-feed price ratio in the current and previous year. The addition of the second milk-feed price variable reduces somewhat the coefficient of the first milk-feed price variable. However, the sum of the elasticities (shown in the column on the right) are higher than for the previous equations. This increase in the elasticity follows the concept of a fan of short-run supply curves. According to this concept elasticities increase as the period allowed for adjustment is extended.

The addition of the milk-feed price ratio lagged two years (Equations 25 and 25A) contributes little to the explanation of changes in milk production. The coefficients for the two year lag are positive but not statistically significant in either time period. The time path of adjustment appears to differ for the two time periods. Major adjustment takes place almost immediately in the 1926-58 period, and after a one year lag in the 1947-58 period. Nevertheless, the major response to the milk-feed price ratio occurs in the first two years.

Equations 26 and 26A include the variable for the beef-milk price ratio with observations lagged one year. Farmers are assumed to respond to a change in this price ratio by varying the number of dairy cows. Dairy cow numbers can be increased by: (1) lowering the culling rate, (2) switching dual purpose animals to milk production, or (3) substituting dairy for beef cows directly. Dairy and beef enterprises are not normally combined on the same farm. Dual purpose herds are in the minority. It was assumed that short-run response to a change in the beef-milk price ratio occurs on the majority of farms through a change in the culling rate. Therefore, the price per cwt. canner and cutter grade cows, Chicago, was used in the beef-milk ratio.

The sign for the coefficient of the lagged beef-milk price ratio should be negative indicating that an increase in the price of beef will be followed by a decline in milk production. Equation 26 shows that although the sign is correct, the coefficient was not statistically significant in the 1926-58 period. In Equation 26A, the elasticity for the beef-milk price ratio is significant. This significance is probably due to the increase in the beef price relative to the milk price in the postwar as compared with the prewar period. Per capita demand for beef increased rapidly in the war and postwar period. Farmers have apparently become more responsive to changes in the beef price. However, the introduction of this variable had a considerable effect on the coefficients of the other variables. For example, the coefficient of the milk-feed price ratio lagged two years is negative.

A sixth variable, the hog-milk price ratio, is added in Equations 27 and 27A. Preliminary empirical estimates indicated that coefficients for the hog-milk price ratio were not significant. Therefore, the observations of this variable were modified to remove the influence of the hog cycle. Cycles in hog production do not appear to have a direct influence on milk production. Milk cow numbers do not vary inversely with hog numbers in the short run. Production per cow may be affected since hogs compete for feed. However, this competition has already been accounted for by including the price of feed. It was stated previously that changes in livestock numbers would affect the farm demand for feed, and hence the price of feed.

The hog-milk price ratio was modified to remove the effect of the hog cycle in the following manner. A variable moving average of hog prices was computed. The number of years in the average was determined by the length of the cycle. The procedure for computing the moving average is described in Appendix C. Figure 4 in Appendix C compares the moving average of the hog price series with the actual observations. The short-run cyclical fluctuations in the price series have been removed by the use of the moving average. Observations of the moving average of prices were divided by the price of milk to obtain the ratios.

Equations 27 and 27A show that this procedure did not give significant results. The coefficient for the hog-milk price ratio in the 1926-58 period is positive when logic indicates it should be negative. The coefficient for the 1947-58 period is negative but not significant.

The addition of the coefficients for the milk price ratios (milk-feed, beef-milk, and hog-milk) gives the sum of the elasticities for milk price in the column to the right of the table. However, the coefficients with incorrect signs (e.g., the hog-milk price in Equation 27 and the milk-feed price ratio lagged two years in Equation 27A) have been omitted from these totals. For each pair of equations in Table 4 (e.g., 23 and 23A) the sum of the elasticities for milk price are very similar. For the fully expanded models (Equations 27 and 27A) the elasticity of milk supply in the short-run (three year period of adjustment) is close to .35. It is predicted that a one percent increase in milk price will be followed by a .35 percent increase in milk production.

The Durbin-Watson d' statistic calculated for the residuals of Equation 27 is 1.159. The test is inconclusive. However, the plotted residuals show a cyclical fluctuation. Therefore, regressions were computed in Table 5 using first differences of logarithms to see if the coefficients or the standard errors of the coefficients differed significantly. When residuals are serially correlated, the tranformation to first differences of logarithms may randomize the error term.

Table 5. Milk production response, Lake States: statistical results using first differences of logarithms showing regression coefficients, standard errors, and R^2 for regression equations of supply response[a]

Equation no.	M/F p. ratio L_{5t}	M/F p. ratio $L_{5(t-1)}$	B/M p. ratio $L_{6(t-1)}$	H/M p. ratio $L_{7'(t-2)}$	R^2	Sum of E for milk
28 1926-58	.15928 (.04512)	.10341 (.03948)	-.02728 (.03145)	.02217 (.03617)	.360	.289
28A 1947-58	.10848 (.04601)	.21748 (.03847)	-.05052 (.02802)	.01684 (.03213)	.844	.361

[a]Abbreviations for column headings: E-elasticity; M/F p.-milk-feed price; B/M p.-beef-milk price; H/M p.-hog-milk price.

The variables in the first-difference equations of Table 5 are the same as those in Table 4, except that the milk-feed price ratio lagged two years has not been included. In most cases, the coefficients and the standard errors of the coefficients in Equations 28 and 28A compare closely with those for Equations 27 and 27A. The most noticeable difference is for the coefficient of the current year's milk-feed price ratio in the 1926-58 period. In the first-difference equation this coefficient was much smaller. The hog-milk price coefficients are both positive. Otherwise, signs conform with logic. The R^2 indicates that a much larger portion of the year to year variance in production has been explained for the postwar years. The sum of the elasticities for milk (omitting hog-milk price coefficients) was higher in the postwar period than for the entire thirty-three years.

Regression analysis using observations of variables in logarithms is continued in Table 6. Two additional variables are included in the equations of this table. The number of variables in a given equation never exceeds six. (In regression analysis it is rare to find as many as six significant variables in a single equation.) These two variables are defined as follows:

L'_{5t-2} = milk-feed price ratio lagged two years, variable moving average. The price per cwt. of milk from combined milk and cream marketings in the Lake States is divided by the price per cwt. of concentrate ration fed to milk cows (average of East North Central and West North Central). Observations are for a variable moving average of this ratio lagged two years.

L_{8t} = yield per acre of hay in the current year as a percentage of the trend in yield. Observations are for the yield per acre of hay in the Lake States divided by the computed values of the linear trend in yield per acre of hay.

The variable moving average of the milk-feed price ratio is computed according to the same procedure used for hog prices explained in Appendix C. Computation of the variable moving average removes short-run year to year fluctuations in the milk-feed price series. The values for the moving average

Table 6. Milk supply response, Lake States, 1926-58 and 1947-58: showing regression coefficients, standard errors, R^2, the sum of the elasticities for milk, and the Durbin-Watson statistic for regression equations using logarithms of observed values.[a]

Equation Number	Time L_{4t}	M/F p. ratio L_{5t}	M/F p. ratio $L_{5(t-1)}$	M/F p. ratio $L_{5'(t-2)}$	B/F p. ratio $L_{6(t-1)}$	H/M p. ratio $L_{7'(t-1)}$	Yield per acre hay L_{8t}	R^2	Sum of E for milk price	D-W stat. d'
(29)	.005646	.14665		.26925				.978	.416	
1926-58	(.0001638)	(.04286)		(.06871)						
(29A)	.006683		.11854	.39194				.963	.510	
1947-58	(.0002673)		(.05172)	(.14907)						
(30)	.005679	.14818		.26507	-.006336			.978	.420	
1926-58	(.0001985)	(.04382)		(.07067)	(.02027)					
(30A)	.006391		.16916	.09292	-.04669			.984	.309	
1947-58	(.0003670)		(.07055)	(.22030)	(.015162)					
(31)	.005673	.157025		.254291	-.006934	.01406		.978	.418	
1926-58	(.0002020)	(.04955)		(.07717)	(.20637)	(.03471)				
(31A)	.006397		.1704	.08645	-.04719	.003251		.984	.304	
1947-58	(.0004021)		(.07757)	(.24938)	(.017387)	(.037679)				
(32)	.005905	.09932		.31278	-.04525	-.01019	.03106	.982	.468	1.307
1926-58	(.0001939)	(.05111)		(.07254)	(.02041)	(.03323)	(.01492)			
(32A)	.006689		.18962	.05048	-.04853	.01883	-.02822	.985	.289	
1947-58	(.0007573)		(.09266)	(.27812)	(.01886)	(.05228)	(.06011)			

[a]Abbreviations for column headings are as follows: p. = price, M/F = Milk/Feed, B/F = Beef/Feed H/M = Hog/Milk, E = elasticity, D-W = Durbin-Watson, stat. = statistic.

have been lagged two years to explain the cow number variations not influenced by short-run changes in the prices of milk and feed. The coefficients and standard errors of this variable are located in Table 6 directly beneath the numerical designation, $L'_{5(t-2)}$.

Equation 29 contains coefficients of the following independent variables: the trend, the milk-feed price ratio in the current year, and the variable moving average of the milk-feed price ratio lagged two years. The milk-feed price ratio in the previous year has been substituted for that of the current year in Equation 29A. Previous empirical estimates have indicated that the production response to a change in the milk-feed price ratio in the postwar years occurred after a lapse of one year. In both equations (Equations 29 and 29A) the elasticities for the variable moving average of the milk-feed price ratio are comparatively high. The sum of the elasticities for milk shown in the column to the right are much higher than previous estimates. This sum exceeds .50 in the postwar period.

The beef-milk price ratio has been included in Equations 30 and 30A. The coefficient for this variable is again significant in the 1947-58 period. However, the inclusion of this variable has had a considerable effect on the coefficients of the other independent variables in the equation. For example, the coefficient of the variable moving average of the milk-feed price ratio declines from .39 in Equation 29A to .09 in Equation 30A. This marked disturbance in the variables suggests multicollinearity in the 1947-58 regression equation. In Equations 31 and 31A the coefficients for the hog-milk price ratio again are positive and not significant in either time period.

The selection of a variable to properly explain changes in milk production due to forage conditions is extremely difficult. This difficulty is due largely to the fact that annual data are used. The forage variable can be handled more adequately when pasture and winter feeding periods are considered separately.

The forage problem is discussed in detail in Appendix D. In brief, the years of good pasture conditions are often accompanied by years of low hay quality. Heavy rainfalls damage hay and delay harvesting operations. The quality is more important than the quantity of hay produced except in years of severe drought. During these years farmers will substitute grain for hay if it is feasible (i.e., if the grain is not also in short supply). Substitution of grain for hay appears to be more prevalent in the summer. In the winter grain is fed according to milk production. Farmers frequently do not increase the feed ration to compensate for poor quality hay.

The complexity of factors involved hinders the selection of a variable which will provide an appropriate explanation of the milk-forage relationship. The hay yield per acre as a percentage of the trend in hay yield was used in Equations 32 and 32A, primarily to account for changes in milk production due to severe drought. Since the current year's hay yield was used, the variable emphasizes pasture conditions in the summer months. Hay and silage inventory adjusted to the calendar year could have been used to indicate the supply of forage available for the winter feeding period. However, as previously stated the quality of harvested forage is frequently a more important factor than the quantity in supply.

Coefficients for hay yield per acre are shown in Table 6 under the numerical designation L_{8t}. The coefficient in the 1926-58 period is positive and statistically significant at the five percent level. Substitution of grain for pasture was not possible in the drought years of the 1930's when both were in short supply. When hay yields fell, milk production fell. The coefficient for hay yield oer acre in Equation 32A is negative but not statistically significant. The negative sign tends to indicate that poor pasture conditions have been more than compensated by an increase in the feeding of grain on pasture during the dry years since the war. However, the sign may reflect in part the fact that hay yield per acre and hay quality are negatively correlated.

With respect to the R^2 the equations of Table 6 are as satisfactory as those of Table 4 (i.e., the R^2 are approximately the same). However, elasticities between the equations of the same time period in Table 6 are less stable. In some cases standard errors are large relative to the coefficients. Thus it appears that less confidence can be placed in the estimates of the individual coefficients in this latter set of equations.

The equations in Table 7 are similar to those in the previous tables in terms of the prices used. The dependent variable and the trend are the same. However, the observations of price variables in these regressions are deflated prices. The variables are defined as follows:

L_{9t} = milk price in the current year deflated. The price per cwt. of milk from combined milk and cream marketings in the Lake States is divided by the U. S. index of prices received for all farm products.

$L_{9(t-1)}$ = milk price in the previous year deflated

L_{10t} = feed price in the current year deflated. The price per cwt. of concentrate ration fed to milk cows (average of East North Central and West North Central) is divided by the U. S. index of prices received for all farm products.

$L_{11(t-1)}$ = beef price in the previous year deflated. The price per cwt. canner and cutter grade cows in Chicago is divided by the U. S. index of prices received for all farm products.

$L'_{12(t-2)}$ = hog price lagged two years deflated, variable moving average. The variable moving average of the price per cwt. of all packer and shipper purchases in Chicago is divided by the U. S. index of prices received for all farm products.

The use of the deflator avoids the assumption that two prices (such as milk and feed in the milk-feed price ratio) have an equal effect upon milk supply. Instead the price variables are entered separately. The observations of each price

Table 7. Milk supply response, Lake States, 1926-58 and 1947-58: showing regression coefficients, standard errors, R^2, the sum of the elasticities for milk, and the Durbin-Watson statistic for regression equations using logarithms of observed values.[a]

Equation Number	Time L_{4t}	Milk p. deflated L_{9t}	Milk p. deflated $L_{9(t-1)}$	Feed p. deflated L_{10t}	Beef p. deflated $L_{11(t-1)}$	Hog p. deflated $L_{12'(t-2)}$	R^2	Sum of E for milk price	D-W stat. d'
(33) 1926-58	.005523 (.0002712)		.24375 (.04531)	-.21973 (.03325)			.978	.244	
(33A) 1947-58	.006897 (.0003238)		.27460 (.02926)	-.07437 (.04071)			.995	.275	
(34) 1926-58	.005562 (.0001320)	.17819 (.04314)	.16076 (.04153)	-.22439 (.02670)			.986	.339	1.912
(34A) 1947-58	.006856 (.0003985)	.01012 (.05517)	.27453 (.02961)	-.07930 (.04920)			.995	.285	
(35) 1926-58	.005590 (.0001710)	.17910 (.04373)	.15966 (.04216)	-.22609 (.02781)	-.004044 (.01514)		.986	.339	
(35A) 1947-58	.006661 (.0002281)	.02389 (.03233)	.25762 (.01801)	-.10474 (.04343)	-.01557 (.008301)		.997	.281	
(36) 1926-58	.005574 (.0001781)	.18255 (.04544)	.15945 (.04282)	-.22473 (.02843)	-.002565 (.01579)	.009601 (.02455)	.986	.342	
(36A) 1947-58	.006726 (.0003300)	.01716 (.04232)	.25974 (.02082)	-.09588 (.04386)	-.01349 (.01143)	.008421 (.02845)	.997	.277	

[a]Abbreviations for column headings are as follows: p. = price, E = elasticity, D-W = Durbin-Watson, stat. = statistic.

series are divided by the deflator. A one to one relationship between the price series and the deflator is assumed (i.e., both have an equal effect on supply). This assumption may not be realistic in all instances. However, if the deflator were included as a separate variable, there would be multicollinearity in the regression equation. Undeflated prices and price indices have trended upward over time and are therefore highly correlated.

The purpose of the deflator is to eliminate the effect of changes in the general price level. It may also reflect movements of the prices of competing products. The index of prices received for all farm products is used as a deflator in the equations of Table 7. However, the index of prices received for livestock might have been a good alternative.

The organization of Table 7 is similar to the previous tables. Regression coefficients are shown together with their standard errors in the column beneath the title of each variable. The number of the equation and time period are indicated in the column to the left.

Equations 33 and 33A contain variables for the trend, the milk price lagged one year, and the feed price in the current year. The low value of the coefficient for the feed price in Equation 33A suggests that it might have been more appropriate to lag the observations of feed price one year in this time period. The lagged milk price accounts for much of the variance in milk production. The coefficient of linear determination for Equation 33A is .995. However, in spite of this high R^2, somewhat less confidence can be placed in the coefficients of the 1947-58 period because of the limited number of observations. The short period of time (only 12 years) increases the chance for spurious correlations.

The milk price in the current year was added in Equations 34 and 34A. The coefficient of this variable was significant in the 1926-58 period, but not in the 1947-58 period. This again suggests that farmers in the postwar period have responded more slowly to a change in the milk-feed price ratio. However, there is no logical explanation for this lagged response.

The addition of beef and hog prices in Equations 35 through 36A contributes little to the explanation of milk supply. As before, beef price coefficients are negative and hog price coefficients are positive, but none of these coefficients are significant.

The sum of the elasticities for milk in the column to the right of Table 7 was obtained by adding the coefficients for deflated milk price (L_{9t} and $L_{9(t-1)}$). For the 1926-58 period the sums compare very closely with Table 4. This indicates that when observations of prices were expressed as ratios the assumption of a one to one relationship was realistic. However, the R^2's for the equations in Table 7 are higher than those for corresponding equations in Table 4.

The Durbin-Watson d' statistic was computed using the residuals in Equation 34, the equation in the 1926-58 period with the lowest error of the estimate. The d' value (1.91) indicates no serial correlation in the residuals.

The emphasis of this study is on the explanation of changes in milk supply. However, it has been stated earlier that explanation and prediction cannot be completely divorced. The ultimate objective in supply analysis is to be able to predict what changes will occur. Thus a model which both "predicts" and "explains" adequately is to be preferred to a model which performs only one of these functions.

Equations 37 and 38A (the equations in each time period with the lowest error of the estimate) were used to predict milk production in 1959. These predictions are shown in Table 8. To the right of the predicted values are the percentage errors for the 1959 estimates. The error of the estimate as a percentage of the mean of the milk production series is shown in the column to the right.

Table 8. Estimate of milk production in the Lake States for 1959: showing observed value, estimated values, percent error of the 1959 estimates, and the error of the estimate as a percentage of the mean.

	Milk production million pounds 1959	Error percent 1959	Error of estimate as percent of mean
Observed value	33,091	-	-
Estimate from Equation 37	32,939	.48	.43
Estimate from Equation 38A	33,291	.60	1.62

The estimate for Equation 37 was below the observed value and the estimate for Equation 38A above. In both cases the error for 1959 was close to one half of one percent. For Equation 38A this error was below the error of the estimate expressed as a percentage of the mean.

The Nerlove Distributed Lags Model

The concept of distributed lags has been employed in previous models of this study using a variable and one or more of its lagged counterparts in the same equation. For example, it was assumed in some models that production response to a change in the milk-feed price ratio occurred over a period of three years (see Equations 25 and 25A). Computation of the long-run elasticity in this manner is theoretically possible but not statistically feasible. In Table 9 and 10 equations have been computed using a modification of the model set forth by Nerlove (50). The procedure for obtaining long and short-run elasticities is illustrated prior to a discussion of the empirical results.

Three milk price variables have been used in each of the equations: the milk-feed price ratio in the current year (L_{5t}), the milk-feed price ratio in the previous year (L_{5t-1}), and the beef-milk price ratio in the previous year (L_{6t-1}). These have been defined on page 81. The milk output in the previous year is also included and is defined as follows:

$L_{1(t-1)}$ = the total milk production in the Lake States in the previous year

The modified Nerlove model is shown in Equation 37. The coefficients used to obtain the long-run elasticity are shown in Equation 38.

(37) $L_{1t}=a+(1-\gamma)L_{1(t-1)}+b_1\gamma L_{5t}+b_2\gamma L_{5(t-1)}+b_3\gamma L_{6(t-1)} \quad 0<\gamma\leq 1$

(38) $$E_L = b_1 + b_2 + b_3$$

where:

$L_{1(t-1)}$ = the production of milk in the previous year

L_{5t} = the milk-feed price ratio in the current year

L_{1t} = the estimated value of milk production in the current year

$L_{6(t-1)}$ = the beef-milk price ratio in the previous year

a = a constant

γ = the coefficient of adjustment

b_1-b_3 = the coefficients of long-run reaction

E_L = the long-run elasticity

The inclusion of two milk-feed price variables (L_{5t} and $L_{5(t-1)}$) combines the procedure used in earlier models of this study with that followed by Nerlove (50). Koyck (40, p. 27-28) illustrates this method of obtaining elasticities. He indicates that any number of lagged values of a given variable may be used in combination with the coefficient of adjustment.

In Table 9 the short-run and long-run elasticities for milk are shown in the two righthand columns. Equations 39 and 39A do not contain a trend variable. The coefficient for lagged milk output incorporates the trend and thus is strongly biased upward. Other elasticities also appear to be biased.

Table 9. Milk supply response, Lake States, 1926-58 and 1947-58: distributed lags model showing regression coefficients, standard errors, R^2, short and long-run elasticities for regression equations using logarithms of observed values.[a]

Equation Number	Milk output $L_{1(t-1)}$	Time L_{4t}	M/F price ratio L_{5t}	M/F price ratio $L_{5(t-1)}$	B/M price ratio $L_{6(t-1)}$	R^2	Short-run E for milk	Long-run E for milk
(39)	.97416		.11637	-.08597	.02877	.973	.116	.228
1926-58	(.03703)		(.03709)	(.03744)	(.02147)			
(39A)	.85001		.22365	.07389	-.004653	.847	.302	1.987
1947-58	(.35413)		(.20285)	(.11684)	(.06018)			
(40)	.16845	.004764	.18407	.05927	-.01526	.987	.259	.310
1926-58	(.16151)	(.000939)	(.02750)	(.04100)	(.01685)			
(40A)	-.24965	.006760	.11760	.21748	-.07636	.993	.411	—
1947-58	(.06045)	(.0002866)	(.022667)	(.014171)	(.022955)			

[a]Abbreviations for column headings are as follows: M/F = Milk/Feed, B/M = Beef/Milk, E = elasticity.

For example, the coefficient for the milk-feed price ratio lagged one year is negative in Equation 39. Neither the long-run nor the short-run elasticities in Equations 39 and 39A seem reasonable.

The inclusion of a trend variable in Equations 40 and 40A raised the R^2. The short-run elasticities appear more logical than those for the two previous equations (Equations 39 and 39A). However, the negative sign for the coefficient of milk output in the previous year makes it impossible to calculate the long-run elasticity in Equation 40A. (The coefficient of adjustment must be less than one as shown in Equation 37.) The lagged milk output in Equations 40 and 40A is highly correlated with time. Multicollinearity may have affected the coefficients.

Regression equations were computed with observations in first differences of logarithms. First differences were used to reduce the problem of multicollinearity by removing the effect of the trend from the series.

Table 10. Milk production response Lake States, 1926-58 and 1947-58: distributed lags model using first differences of logarithms showing regression coefficients, standard errors and R^2 for regression equations of supply response[a]

Equation no.	Milk output $L_{1(t-1)}$	M/F p. ratio L_{5t}	M/F p. ratio $L_{t(t-1)}$	B/M p. ratio $L_{6(t-1)}$	R^2	Short-run E for milk
41	-.10570	.14935	.10859	-.02885	.360	.287
1926-58	(.17389)	(.04363)	(.04226)	(.03184)		
41A	-.31323	.14431	.22242	-.07123	.912	.408
1947-58	(.12907)	(.03427)	(.02895)	(.02264)		

[a]Abbreviations: M/F p.-milk-feed price; B/M-beef-milk.

The estimated values of coefficients for milk-feed and beef-milk price ratio in Table 10 are similar to those for equations using first differences of logarithms in Table 5. However, coefficients for milk production in the previous year (L_{t-1}) are negative in both time periods. The coefficient of adjustment is greater than one. Therefore, the long-run elasticity cannot be computed.

The distributed lag models for milk production in this section contribute very little to an understanding of either short or long-run supply response. The coefficient for lagged milk outputs is particularly sensitive to specification bias. The high R^2 in Equations 40 and 40A indicate that the equations should give reasonable short-run predictions of supply.

The Irreversible Model

The hypothesis that the supply function is not reversible and that the elasticity under rising prices is greater than the elasticity under falling prices is tested in this section. The direction of price movements in both the long and short-run was considered in the selection of observations.

Dairy farmers react to short-run changes in price by changing the feed ration which in turn affects the production per cow. They react to long-run price changes by varying cow numbers. Some farmers go out of business, while those who remain expand herds at a rate in keeping with present returns and long-run prospects. An analysis of reversibility should consider both long-run and short-run adjustments. Price changes were handled in this study in the following manner.

Movements of milk and feed prices were examined in order to divide the thirty-three year period into four groups: (1) long-run rising and short-run rising milk prices, (2) long-run rising and short-run falling milk prices, (3) long-run falling and short-run falling milk prices, and (4) long-run falling and short-run rising milk prices. The exact grouping of years is presented in Appendix E. For the Lake States three periods of long-run price rise and two periods of long-run price decline were defined. A lag of two years was allowed in establishing the boundary between long-run periods. Farmers were assumed to take at least two years to respond to a long-run shift in the direction of price. Short-run price changes were based upon the shift in the milk-feed price ratio in the current year.

The variables used in the equations in Tables 11 and 12 were time and the prices of milk and feed. Inclusion of other price variables would necessitate further subgrouping (e.g., response to milk-feed price changes under rising and falling beef prices). This would reduce the already small number of observations in each category.

In Table 11 observations of milk and feed prices were used as ratios. The column to the right shows the number of observations in each category (i.e., the number of years in each group, the total being thirty-three). The elasticity of the milk-feed price ratio is highest under rising long and

Table 11. Milk supply response, under rising and falling prices, Lake States: showing regression coefficients, standard errors, R^2, and the number of observations for regression equations using logarithms of observed values.[a]

Equation no.	Time L_{4t}	M/F L_{5t}	R^2	No. of observ.
42 Rising long-run Rising short-run	.005677 (.0003895)	.34915 (.13607)	.967	11
43 Rising long-run Falling short-run	.005309 (.0007573)	.23889 (.10781)	.936	9
44 Falling long-run Falling short-run	.005409 (.0002219)	.25370 (.04151)	.990	8
45 Falling long-run Rising short-run	.0044723 (.0003462)	.13385 (.11411)	.992	5

[a]Abbreviations: M/F-milk-feed; observ.-observations.

Table 12. Milk supply response, under rising and falling prices, Lake States: showing regression coefficients, standard errors, R^2, and the number of observations for regression equations using logarithms of observed values.[a]

Equation no.	Time L_{4t}	Milk p. deflated L_{9t}	Feed p. deflated L_{10t}	R^2	No. of observ.
46 Rising long-run Rising short-run	.005583 (.0003467)	.36860 (.11804)	-.30566 (.11440)	.977	11
47 Rising long-run Falling short-run	.005803 (.0004312)	.22654 (.09528)	-.19464 (.04559)	.977	9
48 Falling long-run Falling short-run	.005100 (.0006609)	.16542 (.18265)	-.29301 (.09747)	.990	8
49 Falling long-run Rising short-run	.004701 (.0005111)	.11630 (.18356)	-.12688 (.16865)	.992	5

[a]Abbreviations: p.-price; observ.-observations.

short-run prices. The elasticity under falling long-run but rising short-run prices is lower than the rest.

The difference in elasticities shows that short-run response is affected materially by the direction of short and long-run price movements. The elasticity is higher under rising than under falling long-run prices. However, when short-run prices move in the opposite direction, this tends to "dampen" elasticities of expansion and contraction. For example, the elasticity of the milk-feed price ratio in Equation 43 (rising long-run, rising short-run prices) is lower than that for Equation 42 (rising long-run, falling short-run price).

The equations of Table 12 correspond with those of Table 11 except that the milk and feed prices have been deflated by the index of prices received. This was done to compare the magnitudes of the coefficients for milk and feed to see whether the one to one relationship assumed by the milk-feed price ratio was reasonable. Except in Equation 48 the coefficients compare closely. A rise in the price of milk or a decline in the price of feed will have about the same effect on milk production.

The Recursive Model

The logic of the recursive model for milk production and the computational steps involved have been discussed in the previous section on empirical models. The procedure is reviewed before examining the results for the Lake States. This model provides an estimate of the milk elasticity. In addition, it indicates the relative importance of changes in cow numbers and production per cow in short-run supply response.

Milk production in a given year is determined by the number of cows and the production per cow. Production per cow is affected in the short run by a change in the number of cows. A rapid increase in the number will decrease production per cow as lower quality cows are retained in the herd. Likewise, a rapid decline in cow numbers will increase production per cow as low producers are culled out. Therefore, cow numbers in the current year is included as a variable in the equation explaining production per cow.

Milk production response through a change in cow numbers is normally lagged one or more years. An increase in cow numbers often requires additional fixed investment. Therefore, the variables in an equation explaining cow numbers are predetermined. The regression equation for cow numbers can be computed since it contains only one endogenous or dependent variable. However, the equation for production per cow contains two endogenous variables. Both production per cow and cow numbers are assumed to be correlated with the residual. To eliminate the bias in the coefficients the estimated values for cow numbers in the initial equation should be used in place of observed values of cow numbers in the equation for production per cow. The elasticity for milk is obtained by substituting the righthand side of both equations for cow numbers and production per cow in the identity (milk production equals production per cow multiplied by cow numbers).

In brief, the proper sequence of steps to obtain the price elasticity for milk is as follows: (1) compute the equation with cow numbers as the dependent variable, (2) use the estimated values for milk cow numbers from step one in computing the equation for production per cow, and (3) determine the elasticity for milk production by substituting the results of these equations in the identity.

Table 13 shows the initial step, the solution of the equations with cow numbers as the dependent variable. The independent variables of Equations 50 and 50A are the milk price in the previous year, ($L_{9(t-1)}$), the hog price lagged two years, ($L'_{12(t-2)}$), and the beef cycle in the previous year. The first two variables have been defined on page 94. The latter variable and the dependent variable are defined as follows:

L_{2t} = number of milk cows on farms in the current year, annual average. This is the dependent variable.

$L_{13(t-1)}$ = beef cycle in the previous year. The first difference of the number of cattle and calves kept for beef in the United States, Jan. 1 was lagged one year.

The variable for the beef cycle was used to reflect the complementary relationship between the beef and dairy cattle cycles. Beef cattle cycles have fluctuated more widely than those for dairy cattle. There is still disagreement as to the reason for cyclical cattle movements. Some have felt that they were determined by the available supply of feed; others have considered price relationships a more important factor.

Table 13. Supply response through cow numbers, Lake States, 1926-58 and 1947-58: showing regression coefficients, standard errors, and R^2 for regression equations using logarithms of observed values.[a]

Equation no.	Milk p. deflated $L_9(t-1)$	Hog p. deflated $L_{12}(t-2)$	Beef cycle $L_{13}(t-1)$	R^2
(50) 1926-58	.24857 (.10150)	.11323 (.06437)	.42254 (.13856)	.453
(50A) 1947-58	.27162 (.05045)	.14206 (.10484)	.22465 (.15580)	.643

[a]Abbreviations: p.-price.

The first difference of beef cattle numbers was computed to eliminate the effect of the upward trend in beef cattle. Dairy cattle in the Lake States have shown no similar trend. Because of the regularity of the cattle cycle, taking the first difference does not destroy cyclical fluctuation in the series. However, this procedure has the effect of lagging the cattle cycle since the first difference reflects the rate of change. A cycle represented by the first difference of a series reaches its peak close to the point of inflection of the regular cattle cycle. Thus the first difference cycles move ahead of the regular cycles. On the other hand, the dairy cattle cycles tend to move behind the beef cycles. Lagging the first difference cycle for beef cattle one year gave the most significant results.

The coefficient for milk price is significant in both time periods. The coefficient for the beef cycle is significant in Equation 50, but is much smaller and not significant in Equation 50A. This suggests that the complementary relationship between the cattle cycles has declined in the postwar years. The lagged milk price is significant in both time periods. The positive sign for the hog coefficients suggests that hog and beef cattle numbers have tended to move together. In certain sections of the Lake States, particularly in recent years, this has not been the case. However, the aggregate Lake States figures do not reflect competition between hogs and dairy.

As judged by the low R^2, the results of Equation 50 and 50A are not satisfactory. Because of the failure to obtain a more adequate explanation of cow numbers, the estimated values were not used in solving equations for production per cow. It was felt that the use of these estimated values in preference to actual observations of cow numbers would create an even larger bias.

Production per cow in the Lake States has increased at an increasing rate over the thirty-three year period since 1926. However, the exponential trend used in the equations for milk production (1926 = 1, 1927 = 2, etc.) does not properly describe the growth pattern of production per cow in this period. Figure 7 in Appendix F illustrates this problem. The exponential overestimates the dependent variable in the middle years and underestimates the dependent variable at the extremes. To

Table 14. Supply response through production per cow, Lake States, 1926-58 and 1947-58: showing regression coefficients, standard errors, and R^2 for regression equations using logarithms of observed values.

Equation Number	Time[a] L_{4t}	Milk-Feed price ratio L_{5t}	Milk-Feed price ratio $L_{5(t-1)}$	Milk cow numbers L_{2t}	Hay yield per acre L_{8t}	R^2
(51)	.0001533	.10799	.05267	-.37264	.02887	.973
1926-58	(.00005061)	(.04441)	(.04234)	(.06892)	(.01521)	
(51A)	.006942	.14539	.18619	-.51079	-.03518	.985
1947-58	(.0006712)	(.08038)	(.04938)	(.17317)	(.01040)	

[a]Time entered in Equation 51 as the square of the trend variable: 1926 = 1, 1927 = 4, 1928 = 9 ...

allow for a more rapid rate of growth the trend was calculated by squaring the numbers from one to 33 (i.e., 1926 = 1, 1927 = 4, 1928 = 9...). This procedure was used by Nordin, Judge, and Wahby (53, p. 1004). As Figure 7 shows, the fit is greatly improved.

The variables used in Table 14 to explain production per cow are: time (L_{4t}), the milk-feed price ratio (L_{5t}), the milk-feed price ratio lagged one year (L_{5t-1}), the number of milk cows (L_{2t}), and the hay yield per acre (L_{8t}). These variables have all been defined previously. However, the trend variable, L_{4t}, in Equation 51 has been modified as just described. Cow numbers, L_{2t}, is considered an independent variable. Short-run changes in production per cow can occur through changes in the quantity and quality of the feed ration. The milk-feed price ratio, and hay yield per acre were used to explain changes in the quantity of the ration fed.

The elasticities for the milk-feed price ratio are higher in the 1947-58 period. The coefficient for cow numbers is negative indicating that changes in cow numbers are inversely related to changes in production per cow. The reason for this inverse relationship has been stated previously (i.e., change in cow numbers means a change in the aggregate quality of cows). However, the increase in production per cow will cause the supply function to shift to the right. The supply function (under the effect of an increase in production per cow) could shift more rapidly than the demand function. This persistent increase in production per cow could therefore result in a decrease in the number of cows. This decrease would explain the higher coefficient for cow numbers in Equation 51A.

The final step is to combine the results of elasticities for milk production and production per cow to obtain an elasticity of supply for milk. This is accomplished by substitution into the identity, Equation 52.

Given:

(52) $L_{1t} = (L_{2t})\ (L_{3t})$ (the identity)

(53) $L_2 = a, L_{9t-1}{}^{b_1}, L'_{12t-2}{}^{b_2}, L_{13t-1}{}^{b_3}$ (cow numbers)

(54) $L_3 = c, L_{4t}{}^{d_1}, L_{5t}{}^{d_2}, L_{5t-1}{}^{d_3}, L_{2t}{}^{d_4}, L_{8t}{}^{d_5}$ (production per cow)

where:

L_{1t} = total milk production in farms in the current year

L_{2t} = cow numbers on farms in the current year, annual average

L_{3t} = production per milk cow in the current year

L_{4t} = trend; 1926 = 1

L_{5t} = milk-feed price ratio in the current year

L_{5t-1} = milk-feed price ratio in the previous year

L_{8t} = yield per acre of hay in the current year as a percentage of the trend in yield

L_{9t-1} = milk-price in the previous year deflated by the index of prices received

L_{12t-2} = the variable moving average of the deflated hog price, lagged two years

L_{13t-1} = the first difference of cattle and calves on farms in the United States, Jan. 1

a and c = constants

b_i and d_i = elasticities of coefficients

Equations 53 and 54 are for values in natural numbers although regression equations were computed for observations in logarithms (excepting trend). Substituting Equations 53 and 54 into Equation 52 gives:

$$(55)\quad L_{1t} = \left[a, L_{9t-1}{}^{b1}, L_{12t-2}{}^{b2}, L_{13t-1}{}^{b3}\right] \left[c, L_{4t}{}^{d1}, L_{5t}{}^{d2}, L_{5t-1}{}^{d3}, L_{2t}{}^{d4}, L_{8t}{}^{d5}\right]$$

The next step is to substitute the righthand side of Equation 53 for L_{2t} in Equation 55.

$$(56)\quad L_{1t} = \left[a, L_{9t-1}{}^{b1}, L_{12t-2}{}^{b2}, L_{13t-1}{}^{b3}\right] \left[c, L_{4t}{}^{d1}, L_{5t}{}^{d2}, L_{5t-1}{}^{d3}(a, L_{9t-1}{}^{b1}, L_{12t-2}{}^{b2}, L_{13t-1}{}^{b3})^{d4}, L_{8t}{}^{d5}\right]$$

By collecting terms, Equation 56 may be written as follows:

$$(57)\quad L_{1t} = \left[a, L_{9t-1}{}^{b1}, L_{12t-2}{}^{b2}, L_{13t-1}{}^{b3}\right]^{1+d4} \left[c, L_{4t}{}^{d1}, L_{5t}{}^{d2}, L_{5t-1}{}^{d3}, L_{8t}{}^{d5}\right]$$

The variables which contain the prices of milk are L_{9t-1}, L_{5t}, and L_{5t-1}. The coefficients of these variables are their exponents, $b_1(1+d_4)$, d_2, and d_3. The elasticity for milk (E_m) <u>ceteris paribus</u>, is the sum of the milk price elasticities.

$$(58)\qquad E_m = b_1(1+d_4) + d_2 + d_3$$

The symbol b_1 can be identified in Table 13 as the coefficient of the milk price in the previous year. The symbols d_2, d_3, and d_4, are the coefficients in Table 14 for the milk-feed price ratio in the current and previous year, and for cow numbers.

The short-run elasticity for milk is obtained by substituting the coefficients from these tables into Equation 58.
For the 1926-58 period the elasticity is:

(59) $$E_m = .107 + .053 + (.249)(.627) = .316$$

For the 1947-58 period the elasticity is:

(60) $$E_m = .145 + .186 + (.272)(.489) = .464$$

The short-run elasticity for the 1926-58 period compares closely with elasticities obtained using single equations. The estimated elasticity in the postwar period is somewhat higher than previous results.

Summary

Several models were used in this section to determine the short-run elasticity of response to milk. The variables used have been principally trend, the price of milk, the price of feed, and the prices of competing livestock products. The period allowed for adjustment varied from one to three years.

The results indicate that as the period of adjustment is extended the short-run elasticity of supply increases. This supports the concept of a distributed lag. Response to a change in the price of milk occurs over several time periods. With a three year period of adjustment the short-run elasticity of supply appears to be between .30 and .35. Some of the estimates fell outside of this range. Nevertheless, there is no significant difference in elasticities between the two time periods.

The variables for trend, milk price, and feed price explained most of the variance in milk supply. Addition of variables for beef and hogs contributed little to the explanation of production response. However, estimates of the beef price coefficient were significant in the postwar period. The competition from livestock products may be accounted for, in part, through changes in the price of feed.

Nerlove models added little to the understanding of either the short or the long-run elasticity of supply. However, the irreversible model showed that the elasticity was higher under rising than under falling prices. Elasticities were higher when short and long-run prices moved in the same direction. Opposing short and long-run price changes decreased the elasticity of response.

Finally, the recursive model indicates that the influence of changes in cow numbers and production per cow on short-run changes in supply have been nearly equal. However, the changes in cow numbers have not been adequately explained. Consequently, more confidence can be placed in the elasticities for milk supply obtained through the single equation models.

SUPPLY RESPONSE IN THE NORTHEAST

Estimates of short-run supply response were obtained for the Northeast following the procedure of the previous section. Regression equations were computed using time series data principally for two periods, 1926-58 and 1947-58. Results using single equation and recursive models are presented in equation and tabular form. The basic data used in computing equations are presented in Tables 41 and 42 of Appendix A.

Estimates of supply response obtained for the Lake States using the Nerlove model for distributed lags are shown in Tables 9 and 10 of the previous section. These results indicate that the Nerlove model does not provide dependable estimates of either short or long-run supply elasticities. Coefficients are biased either due to failure to include a trend or due to multicollinearity arising from the inclusion of the trend variable. In either case, the bias occurs because of the trend in the dependent variable and in its lagged counterpart. Milk production has shown an upward trend in both the Northeast and California. There is, therefore, no reason to believe that unbiased elasticities can be obtained in either of these regions. Coefficients have not been obtained in this section using the Nerlove model.

Many of the independent variables used in the Northeast correspond to those used in the Lake States (although observations of variables are not the same). However, the alternative uses for farm resources are limited in the Northeast.

Therefore, in the selection of variables greater emphasis has been placed upon prices which reflect the cost of dairy production. In addition to the price of feed, an index of farm wage rates has been used in several of the regressions. The price of hogs has not been used as a variable. On the other hand, the beef price has been used since farmers can respond to fluctuation in beef price by changing dairy cow numbers.

Conventional Single Equations

Tables 15 through 18 contain coefficients estimated by regressions with milk production as the dependent variable. The tables were constructed following the pattern used in the Lake States. Since these tables are complex and contain many figures, their organization is reviewed.

Titles of the variables are shown at the top of each column together with the numerical designation. (In this section N stands for Northeast.) The variables are defined in the text. The lefthand column indicates the equation number and the time period. Beneath the coefficients of each equation in parantheses are their standard errors. The R^2 in the first column to the right of the coefficients indicates the percentage of the variance in milk production explained by the independent variables of the equation. The sum of the elasticities for milk is the absolute sum of the coefficients of variables containing a milk price (e.g., milk-feed price ratio, beef-milk price ratio). Variables whose signs are not in accord with logic have not been included in this sum. In Tables

15 and 17 the Durbin-Watson d' statistic was computed for the equation in the 1926-58 period with the lowest error of the estimate.

An initial problem encountered in the Northeast analysis was the fitting of the trend variable. Preliminary computations used an exponential function for trend with 1926 = 1, 1927 = 2 etc. The values for trend were not converted to logarithms in computing the regression equations. The elasticities for the milk-feed price ratio using this trend were negative. As in the case of production per cow in the Lake States this form of the exponential trend did not accurately explain production growth. The nature of the problem is shown in Figure 9, Appendix F.

Figure 9 was constructed in the following manner. First the exponential trend (1926 = 1, 1927 = 2 etc.) was fitted to the 1926-58 series for milk production and the milk-feed price ratio. Estimated values of the dependent variable (N_{1t}) in each year were obtained by substituting observations of time for N_{4t} in Equation 61. Deviations from the trend were

$$\log N_{1t} = \log a + \log b(N_{4t}) \tag{61}$$

where:

N_{1t} = the log of the expected milk output in time period t

N_{4t} = time: 1926 = 1, 1927 = 2

log a = a constant

log b = a coefficient of time

obtained by subtracting these estimated values from observed values in the same year. The procedure was repeated with the milk-feed price ratio (N_{5t}) as the dependent variable in Equation 61. Using deviations from the trend, points were plotted for price against quantity. The milk-feed price ratio is on the vertical axis, and the quantity of milk on the horizontal axis. For example, the deviations from the trend for the milk-feed price ratio was one in 1926. The deviation from the trend for milk production was 918 (million lbs.) in the same year. The intersection of these two points is represented on the scatter diagram by a dot which is identified by the number 26 (i.e., 1926). In Figure 9 observations for the 1920's and 1950's tend to be grouped toward the right and observations for the mid-1940's toward the left. Thus, deviations from the milk production trend are for the most part greater than zero for the former group of years (the 1920's and the 1950's) and less than zero for the latter group of years (the mid-1940's).

Squared values of the numbers from one to 33 (1926 = 1, 1927 = 4 etc.) were used to fit a second trend. Squaring the observations of time has the effect of increasing the rate of growth in the later years relative to the earlier years. Deviations from this time trend are plotted in Figure 10 with the milk-feed price ratio on the vertical axis and milk production on the horizontal axis. The observations are not as widely scattered as in Figure 9 indicating that the fit of the trend function has been improved. In Tables 15 through 18 all of the regression equations for the 1926-58 period have been computed using this modified form of the exponential function for the trend variable.

The variables used in the equations shown in Table 15 are defined as follows:

N_{1t} = total milk production on farms in the current year. This is the dependent variable.

N_{4t} = time: 1926 = 1, 1927 = 2, 1928 = 3...

$N_{4't}$ = time: 1926 = 1, 1927 = 4, 1928 = 9...

N_{5t} = the milk-feed price ratio in the current year. The price per cwt. of milk from combined milk and cream marketings in the Northeast is divided by the price per cwt. of concentrate ration fed to milk cows in the Northeast.

N_{5t-1} = milk-feed price ratio in the previous year

N_{5t-2} = milk-feed price ratio two years previous

N_{6t-1} = beef-milk price ratio in the previous year. The average price per cwt. received by farmers for beef cattle in the Middle Atlantic states is divided by the price per cwt. of milk from combined milk and cream marketings in the Northeast.

N_{14t-1} = milk-wage price ratio in the previous year. The price per cwt. of milk from combined milk and cream marketings in the Northeast is divided by an average of the index of composite wage rates for New England and the Middle Atlantic states.

Equations 62 and 62A show coefficients for trend and the milk-feed price ratio in the current year. In both time periods more than 90 percent of the variance in milk production is explained by trend. The coefficient for the milk-feed price ratio in the 1926-58 period is positive but not signifi-

Table 15. Milk supply response, Northeast, 1926-58 and 1947-58: showing regression coefficients, standard errors, R^2, the sum of the elasticities for milk, and the Durbin-Watson statistic for regression equations using logarithms of observed values.[a]

Equation Number	Time[b] N_{4t}	Milk-feed p. ratio N_{5t}	Milk-feed p. ratio $N_{5(t-1)}$	Milk-feed p. ratio $N_{5(t-2)}$	Beef-milk p. ratio $N_{6(t-1)}$	Milk-wage p. ratio $N_{14(t-1)}$	R^2	Sum of E for milk p.	D-W stat. d'
(62)	.0001562	.09474					.976	.095	
1926-58	(.000004240)	(.06124)							
(62A)	.008291	-.10882					.929	-	
1947-58	(.0007728)	(.13875)							
(63)	.0001563	.08914	.01051				.976	.100	
1926-58	(.000004691)	(.07236)	(.06903)						
(63A)	.008646	-.11027	-.08022				.932	-	
1947-58	(.0009858)	(.17889)	(.15109)						
(64)	.0001560	.08726	.04226	-.04962			.976	.130	
1926-58	(.000004732)	(.07290)	(.08139)	(.06617)					
(64A)	.007946	-.15026	.06859	-.27083			.957	.069	
1947-58	(.0009846)	(.16633)	(.16090)	(.14659)					
(65)	.0001527	.09926	.02626		.02124		.977	.126	
1926-58	(.000005509)	(.07126)	(.06884)		(.01847)				
(65A)	.008553	-.11374	-.09020		-.01165		.933	.012	
1947-58	(.001076)	(.18977)	(.16347)		(.03254)				
(66)	.001236	.06469	.08932		.02277	-.10496	.981	.154	1.160
1926-58	(.00001229)	(.06734)	(.06822)		(.01712)	(.04029)			
(66A)	.001473	.01593	.04789		.002243	-.49111	.989	.064	
1947-58	(.001897)	(.08720)	(.07616)		(.01461)	(.08996)			

[a]Abbreviations for column headings are as follows: p. = price, E = elasticity, D-W = Durbin-Watson, stat. = statistic.

[b]Time entered in Equations 62, 63, 64, 65, and 66 as the square of the trend variable: 1926 = 1, 1927 = 4, 1928 = 9....

cant at the five percent level. However, the coefficient for the milk-feed price ratio in the postwar period is negative.

The milk-feed price ratio in the previous year has been added in Equations 63 and 63A. The coefficients of this variable explain very little of the variance in milk production. The coefficients for the lagged milk-feed price ratio in the 1947-58 period is negative. Coefficients for the milk-feed price ratio lagged two years (Equations 64 and 64A) are negative in both time periods. Therefore, these variables were omitted when the model was expanded to include two additional variables.

Observations of the beef-milk price ratio in the previous year were used in Equations 65 and 65A to explain short-run changes in milk production due to changes in culling rates. The number of beef cattle or dual purpose animals in the Northeast is very small and the direct competition from beef cattle production of minor importance. The beef price used in the ratio is the average price per cwt. received for beef cattle in the Middle Atlantic states. A series of prices of cows for slaughter was not available for the Northeast for the thirty-three year period. However, slaughter cattle and cull cow prices are known to be closely correlated. In fact, a portion of the beef cattle price is represented by the cull cow price. Therefore, the use of a beef cattle price to reflect changes in culling rates seems appropriate.

The coefficients for the beef-milk price ratio in the previous year are not significant in either time period. The positive sign in the 1926-58 period is contrary to logic. A positive sign implies that an increase in the price of beef relative to the price of milk will results in an increase in milk production in the following year.

Coefficients for the lagged milk-wage price ratio are shown in Equations 66 and 66A at the bottom of the table beneath the numerical designation $N_{14}(t-1)$. The wage index used in the series is based upon a weighted average of monthly and hourly payments to hired help. It is a matter of controversy whether labor is forced off the farm by farmers refusing to pay higher wage rates or encouraged off by increased urban opportunities. (This pressure exerted on farm labor from two directions is referred to be some as a "push" or a "pull.") However, this wage index is assumed to reflect both forces (i.e., changes in urban and farm wage rates are assumed to be correlated).

There are three problems concerning the interpretation of the coefficients of this variable. First, it is difficult to know whether a high wage rate will increase or decrease production in the short-run. Less labor could mean fewer cows and lower production. Recent experience has shown that in the long run substitution of capital for labor in the form of labor saving devices will more than compensate for the reduction in labor. The time needed for this adjustment is not known.

Secondly, wage rates are closely correlated with consumer income. Hence the wage rate, even though lagged, may partly explain the demand for dairy products. The income elasticity of demand for dairy products has declined in recent years. Finally, use of the milk-wage price ratio may lead to multicollinearity in the regression equation. Wage rates have increased more rapidly over time than milk prices. The milk-wage price ratio trends downward and is negatively correlated with time (N_{4t}). It was decided to use the milk-wage price ratio in spite of these difficulties to see if this variable gave a significant explanation of the variance in production.

The coefficient for the milk-wage price ratio is negative and significant in both time periods. However, introducing this variable has appreciably changed the values of many of the other coefficients in Equations 66 and 66A. In the latter equation, trend is no longer significant, and the coefficients for the milk-feed price ratio which were formerly negative are now positive. Although the R^2 is .989, the only significant variable in the equation is the milk-wage price ratio.

It appears that the regression coefficients have been affected by multicollinearity. Methods of identifying and handling problems of multicollinearity are not entirely satisfactory. Tests have been devised, but a common practice is to set an upper limit for the correlation between two independent variables (e.g., .70). When the correlation exceeds this arbitrary limit the variables are not included in the same equation.

No fixed limit has been set in this study. By expanding models through the successive addition of variables the effect of multicollinearity can often be detected in the coefficients. The use of different forms of the equation (e.g., observations in logarithms and in first differences of logarithms) provides a further check. Radical change in coefficients suggests the presence of multicollinearity. However, this procedure, like the others used, is not entirely free from error.

The sum of the elasticities for milk (in the column to the right of the coefficients) was obtained by adding the absolute values of milk price coefficients (e.g., coefficients for milk-feed and beef-milk price ratios) whose signs were consistent with logic. The coefficients for the milk-wage price ratios in Equations 66 and 66A have been omitted from the sums. The milk-wage price ratio appears to have explained at least in part, the trend in milk production. A dash appears in the column for two of the equations (Equations 64A and 65A) because none of the signs of the milk price coefficients were "correct." The highest elasticity obtained from the summation of milk price coefficients is .154 (see Equation 66). The sum of the elasticities is higher in the 1926-58 period than in the postwar years (comparing equations with the same variables in each time period).

In spite of the fact that the coefficients of linear determination are comparatively high, little confidence can be placed in the estimates of price elasticities shown in Table

15. Standard errors of coefficients are high. Signs for many of the coefficients in the 1947-58 period are inconsistent with logic. These "wrong" signs could be due to, (1) errors of observation, (2) omission of relevant variables, or (3) spurious correlations. The chance of bias due to any one of these causes is increased by the small number of observations and the comparative stability of milk production in the region. The data were re-examined to determine why incorrect signs were obtained and whether or not these could be adequately explained.

Figures 9 and 10 in Appendix F, as previously noted, show the plottings of price against quantity when the effect of trend has been removed. Observations represent the deviations from the squared values of time (1926 = 1, 1927 = 4 etc.). These diagrams have thus taken into account the effect on production of three important variables - trend and the prices of milk and feed. In both diagrams observations for the years 1957 and 1958 lie above and to the left of other postwar observations. This situation suggests a low production response for the years in question in spite of a favorable milk-feed price ratio. Examination of the trends in production and cow numbers (Figures 1, 2, and 3) indicates that a decline in production in 1957 was due to a decrease in cow numbers. The decline in dairy cow numbers was apparently related through the beef cycle to a decline in the beef price in 1953 and 1954.

This explanation indicates the complexity of the beef-milk production relationship. Short-run changes in dairy cow numbers are assumed to be related to changes in culling rates. Thus, when beef prices increase relative to milk prices, milk production should decline. On the other hand, longer run changes in dairy cow numbers are related to the beef cattle cycle. An increase in beef prices is followed by an increase in both beef and dairy cow numbers. The lag in response is three to four years or longer because of the long period of growth and reproduction. This last relationship is complementary rather than competitive (i.e., an increase in beef price is followed by an increase in milk production and vice versa). The nature of this complementary relationship between cycles is not fully understood. However, cycles have been more closely correlated in prewar than in postwar years.

The discussion suggests that the negative price elasticities obtained for the milk-feed price ratio and the positive elasticities obtained for the beef-milk price ratio in many of the equations of Table 15 may have been due to specification bias. The change in cow numbers has not been adequately explained by the beef-milk price ratio in the previous year (see Equations 65A and 66A). An attempt was made to explain the decline in cow numbers and hence in milk production in 1957 and 1958 by lagging the beef price up to four years. This procedure proved unsuccessful. A less satisfactory procedure was to omit the observations for milk production in the years 1957 and 1958. Coefficients were obtained for both

Table 16. Milk supply response, Northeast, 1926-56 and 1947-56: showing regression coefficients, standard errors, R^2, and the sum of the elasticities for milk for regression equations using logarithms of observed values.[a]

Equation Number	Time N_{4t}	Milk-feed price ratio N_{5t}	Milk-feed price ratio $N_{5(t-1)}$	Beef-milk price ratio $N_{6(t-1)}$	Milk-wage price ratio $N_{14(t-1)}$	R^2	Sum of E for milk price
(67) 1926-56	.0001691 (.000005850)	.15433 (.05853)	.03582 (.05530)	-.004672 (.01595)		.982	.195
(67A) 1947-56	.009564 (.0008215)	.11779 (.15763)	.16734 (.16952)	-.02408 (.02281)		.959	.309
(68) 1926-56	.0001394 (.000009694)	.11777 (.05139)	.10188 (.05104)	-.002960 (.01373)	-.10672 (.02982)	.991	.223
(68A) 1947-56	.0002844 (.001993)	.05683 (.07901)	.13192 (.07249)	-.002433 (.01217)	-.42419 (.08921)	.991	.191

[a]Abbreviation for column heading is as follows: E = elasticity.

time periods omitting these observations.

Equations 67 and 68 in Table 16 correspond with Equations 65 and 66 in the preceding table in terms of the variables used. These variables are defined on page 121. The omission of these two observations shows the following effect. First, the signs of the equations in Table 16 are consistent with logic. Coefficients for the milk-feed price ratio are positive in both time periods. Coefficients for the beef-milk price ratio are negative, although not significant. Secondly, standard errors of the coefficients are lower indicating that greater confidence can be placed in the values. Finally, the higher R^2's show that more of the variance in production has been explained.

The coefficients for the milk-feed price ratio in the postwar period were altered significantly by the omission of observations. In Equation 67A, for example, the elasticities for the milk-feed price ratio in the current and the previous year are .118 and .167 respectively. Coefficients for the corresponding equation in Table 15 are -.114 and -.090. The sum of the elasticities for milk price (milk-feed plus beef-milk price coefficients) in the 1926-58 period is close to .20. In the 1947-58 period the sum of the elasticities is .309 in Equation 67A, but only .191 in the following equation. These sums of elasticities are higher than those for equations in Table 15.

The coefficients in Table 17 were obtained using observations of deflated prices. The assumption that farmers react equally to a change in the price of milk or feed appeared to be realistic in the Lake States. However, in the Northeast where the bulk of concentrate is purchased and competition for feed grain is of lesser importance, this relationship might differ.

Table 17 contains coefficients of regressions computed for four time periods, 1926-58, 1947-58, and 1926-56, 1947-56. The last two equations in the table were computed for these shortened time series. The coefficients obtained omitting the 1957-58 observations can be compared with the corresponding coefficients for the other equations.

The variables used in Table 17 are defined as follows:

N_{9t} = the price of milk in the current year deflated. The price per cwt. of milk from combined milk and cream marketings in the Northeast is divided by the U. S. index of prices received for all farm products.

N_{9t-1} = the price of milk in the previous year deflated

N_{10t} = the price of feed in the current year deflated. The price per cwt. of concentrate fed to milk cows in the Northeast is divided by the U. S. index of prices received for all farm products.

N_{11t-1} = the price of beef in the previous year deflated. The price per cwt. of beef cattle in the Middle Atlantic states is divided by the U. S. index of prices received for all farm products.

N_{15t-1} = the wage index in the previous year deflated. The index of composite wage rates averaged for New England and the Middle Atlantic states is divided by the U. S. index of prices paid for agricultural production.

The trend variables, N_{4t} and N'_{4t} have been defined on

Table 17. Milk supply response, Northeast, 1926-58, 1947-58 and 1926-56, 1947-56: showing regression coefficients, standard errors, R^2, the sum of the elasticities for milk and the Durbin-Watson statistic for regression equations using logarithms of observed values.[a]

Equation Number	Time[b] N_{4t}	Milk p. deflated N_{9t}	Milk p. deflated $N_{9(t-1)}$	Feed p. deflated N_{10t}	Beef p. deflated $N_{11(t-1)}$	Wage index deflated $N_{15(t-1)}$	R^2	Sum of E for milk price	D-W stat. d'
(69)	.0001511	.09969	.01648	-.15810	.02457		.978	.116	
1926-58	(.000006068)	(.07493)	(.04684)	(.07925)	(.02069)				
(69A)	.008314	-.10300	.07944	.13454	.01667		.929	.079	
1947-58	(.001162)	(.23204)	(.15677)	(.23693)	(.06173)				
(70)	.0001468	.09694	.02262	-.16105	.02587	.01953	.979	.120	1.125
1926-58	(.000008522)	(.07568)	(.04799)	(.08005)	(.02095)	(.02679)			
(70A)	.007239	.04499	.03626	-.02560	.01304	.21793	.940	.081	
1947-58	(.001639)	(.28220)	(.16484)	(.29369)	(.06245)	(.23180)			
(71)	.0001669	.13196	.03918	-.18379	.0008097		.983	.171	1.510
1926-56	(.000006700)	(.06567)	(.04045)	(.06731)	(.01863)				
(71A)	.01020	-.02795	.31873	.006044	.04121		.989	.319	
1947-56	(.0006125)	(.12314)	(.08105)	(.11302)	(.02668)				

[a]Abbreviations for column headings are as follows: p. = price, E = elasticity, D-W = Durbin-Watson, stat. = statistic.

[b]Time entered in Equations 69, 70, and 71 as the square of the trend variable: 1926 = 1, 1927 = 4, 1928 = 9....

page 121. The latter variable is used in equations for the 1926-58 and 1926-56 periods. The wage index has been deflated by the index of prices paid for production. This ratio reflects the cost of labor as opposed to the cost of other items in production. The index of prices paid for production does not include interest, taxes, and wage rates. Although the feed price has been deflated by the index of prices received, using an index of prices paid might be justified. Feed represents both a cost and a return in the farm economy.

Equations 69 and 69A contain variables for trend (N_{4t}), the price of milk in the current and in the previous year (N_{9t} and N_{9t-1}), the price of feed in the current year (N_{10t}) and the price of beef in the previous year (N_{11t-1}). With the exception of the sign of the coefficient for beef price, which is positive but not significant, the signs in the 1926-58 period conform with logic. However, for the postwar period results are less satisfactory. Coefficients for milk price in the current year, and for feed and beef prices have the "wrong" sign. The R^2 is much lower than for the previous equation. Standard errors of the regression coefficients are comparatively high.

Coefficients for the milk-wage price ratio are not significant in Equations 70 and 70A. However, the positive sign for the coefficient suggests that when labor costs increase relative to other production costs, the substitution of capital for labor will increase milk production.

The omission of observations for the dependent variable in 1957 and 1958 (Equations 71 and 71A) increases the R^2 and reduces the standard errors of the coefficients particularly in the 1947-58 period. Standard errors of coefficients in Equation 71A are approximately one-half of those for corresponding coefficients in Equations 69A and 70A. For example, the standard error of the milk-feed price ratio in the previous year (N_{9t-1}) is .165 in Equation 70A, but only .081 in Equation 71A. However, three of the coefficients in this latter equation (milk and feed price in the current year, and beef price in the previous year) still have "wrong" signs. The most important price variable explaining changes in milk production is the milk-feed price ratio in the previous year. The sum of the coefficients for milk price in Equation 71A is almost double that for the 1926-58 period.

In summary, the results obtained by omitting observations of milk production in the last two years are not completely satisfactory. However, estimates of elasticities for the price of milk are statistically more reliable and in keeping with the a priori assumption that the elasticity of response has increased in the postwar period.

Durbin-Watson statistics were computed using residuals from Equations 70 and 71. The tests were inconclusive for both equations. However, the d' statistic is somewhat lower for Equation 70. Observations of the variables in Equations 70 and 70A were transformed to first differences of logar-

Table 18. Milk supply response, Northeast, 1926-58 and 1947-58: showing regression coefficients, standard errors, R^2, and the sum of the elasticities for milk price for regression equations using first differences of logarithms.[a]

Equation Number	Milk price deflated N_{9t}	Milk price deflated $N_{9(t-1)}$	Feed price deflated N_{10t}	Beef price deflated $N_{11(t-1)}$	Wage index deflated $N_{15(t-1)}$	R^2	Sum of E for milk price
(72) 1926-58	-.09952 (.04860)	.20548 (.06839)	-.07325 (.02764)	.03308 (.02851)	.21404 (.08635)	.454	.205
(72A) 1947-58	-.01254 (.13133)	.29942 (.19613)	.05209 (.19453)	.02236 (.07359)	-.04557 (.13841)	.455	.299

[a]Abbreviation for column heading is as follows: E = elasticity.

ithms. The coefficients obtained with observations as first differences are shown in Table 18.

The signs for milk price in the current year are negative in both time periods. The milk price in the previous year gives the best explanation of year to year change in milk production. Coefficients for the feed price and the wage index are also significant in the 1926-58 period. The sign for the coefficient of the wage index is positive. In the 1947-58 period, however, the sign is negative and the coefficient is not significant. One explanation for this difference between time periods might be as follows. Wage rates are positively correlated with consumer income. The wage rate may therefore give a partial explanation of changes in demand for dairy products. Under this assumption, the decline in the income elasticity of demand for dairy products would be responsible for the insignificant coefficient in the postwar years.

Transformation to first differences did not bring about a significant change in the standard errors of the regression coefficients. However, there is a noticeable difference in the magnitudes of some of the coefficients. The sum of the elasticities for milk price conforms more closely with the values obtained in the previous table when the 1957 and 1958 observations were omitted (see Equations 71 and 71A). The elasticity for milk price in the postwar period is appreciably higher than that for the entire thirty-three years.

Those equations in each time period with the lowest error of the estimate were used to predict milk production in 1959. Equations for four time periods were used (1926-58, 1947-58, 1926-56, and 1947-56). These equations are identified in the lefthand column of Table 19 and appear in Tables 15 and 16. The predicted values are shown to the right of the equation number. The error of the estimate expressed as a percentage of the mean and the 1959 percentage error associated with each of these equations is also shown.

Table 19. Estimate of milk production in the Northeast for 1959: showing observed value, estimated values, percent error of the 1959 estimate, and the error of the estimate as a percentage of the mean.

	Milk production million lbs. 1959	Error percent	Error of estimate as percent of mean
Observed value	23,863	-	-
Estimate from Equation 66	25,317	6.09	2.80
Estimate from Equation 66A	23,906	.18	1.29
Estimate from Equation 68	25,710	7.74	2.10
Estimate from Equation 68A	25,293	6.00	1.02

The predicted values from all four equations overestimated the observed value. Equation 66A provided the most accurate estimate in spite of the fact that some of the coefficients in the equation have little economic significance. Errors in prediction for the other three equations were comparatively large. These large errors emphasize the danger of (1) projecting estimates as much as three years, (2) extrapolating with an exponential trend. The largest error was obtained for Equation 68 (1926-56 time period). The danger of extrapolation is increased when observations of time have been squared (Equations 66 and 68).

The Irreversible Model

The same procedure was used in the Northeast as in the Lake States for testing the hypothesis that the elasticity of expansion exceeds the elasticity of contraction. The division of years according to four categories of long and short-run

Table 20. Milk supply response under rising and falling prices, Northeast: showing regression coefficients, standard errors, R^2, and the number of observations for regression equations using logarithms of observed values.[a]

Equation no.	Time N'_{4t}	M/F p. N_{5t}	R^2	No. of observ.
(73)				
Rising long-run	.005130	-.14044	.969	15
Rising short-run	(.0002763)	(.11329)		
(73A)				
Rising long-run	.005921	-.32210	.975	6
Falling short-run	(.0005829)	(.23279)		
(74)				
Falling long-run	.006205	.16580	.990	8
Falling short-run	(.0003180)	(.09354)		
(74A)				
Falling long-run	.007955	.67352	.997	4
Rising short-run	(.0007620)	(.19838)		

[a]Abbreviations: M/F p.-milk-feed price; observ.-observations.

price changes is shown in Appendix E. The results are shown in Table 20. The independent variables of the regression equations were time (N'_{4t}) and the milk-feed price ratio in the current year (N_{5t}). The number of observations in each category is shown in the righthand column of the table.

Coefficients for the milk-feed price ratio are negative during periods of long-run rising price and positive during periods of long-run falling price. Three of the four coefficients are not significant. Only the elasticity in Equation 74 could be considered reasonable in terms of previous results.

It is difficult to accurately estimate price elasticities when (1) production is comparatively stable, and (2) the number of observations in the equation is very small. Under these conditions errors in observation or in model specification assume greater importance. The inability to obtain an adequate explanation of production response in the Northeast under rising and falling prices appears to be due, at least in part, to the two factors mentioned above.

The Recursive Model

The recursive model for milk production is based upon the assumption that (1) the number of cows in the current year is explained by predetermined variables, and (2) the number of cows in turn helps to explain production per cow in the current year. Elasticities for milk price obtained using cow numbers and production per cow in turn as the dependent variable can be combined to estimate the elasticity for milk production. The nature of the recursive model has been explained in greater detail in the two previous sections.

The first step in the recursive solution is to compute coefficients for equations with cow numbers as the dependent variable. The results of regressions for the two time periods 1926-58 and 1947-58 are shown in Table 21. The variables for time (N_{4t}) and for the beef-milk price ratio in the previous year (N_{6t-1}) have been defined previously. Additional variables used in these equations are defined as follows:

N_{2t} = the number of cows on farms in the current year, annual average. This is the dependent variable.

$N_{5'{t-2}}$ = milk-feed price ratio lagged two years, variable moving average. The price per cwt. of milk from combined milk and cream marketings in the Northeast is divided by the price per cwt. of concentrate ration fed to milk cows in the Northeast. Observations are for a variable moving average of this ratio lagged two years.

N_{13t-1} = the beef cycle in the previous year. The first difference of the number of cattle and calves kept for beef in the United States, Jan. 1 was lagged one year.

The trend variable is used in the 1926-58 period to explain the gradual increase in cow numbers in the Northeast. This increase has occurred even though farm numbers have declined and land has been retired from agriculture. The increase has been made possible by greater labor efficiency.

The variable moving average of the milk-feed price ratio was computed according to the procedure shown in Appendix C.

Table 21. Supply response through cow numbers, Northeast, 1926-58 and 1947-58: showing regression coefficients, standard errors, and R^2 for regression equations using logarithms of observed values.[a]

Equation no.	Time N_{4t}	M/F p. ratio N'_{5t-2}	B/M p. ratio N_{6t-1}	Beef cycle N_{13t-1}	R^2
(75) 1926-58	.0008953 (.0001868)	.16089 (.06726)	-.02541 (.01645)	.18059 (.05687)	.720
(75A) 1947-58	-	-.14653 (.07129)	-.08044 (.01694)	.27413 (.07024)	.832

[a]Abbreviations: M/F p.-milk-feed price; B/M p.-beef-milk price.

The purpose of using the variable moving average was to remove short-run fluctuations in the series which were assumed not to affect cow numbers. The length of the average for each year depended on the length of the short-run cycle.

The nature of the beef-milk relationship has been discussed in connection with the single equation models of this section. This relationship was not adequately explained in these models by the lagged beef price. Signs were positive and not significant. The attempt to explain a decrease in milk production (which was apparently related through the beef price to a decline in dairy cow numbers) was unsuccessful. This failure led to the computation of equations omitting observations for the years 1957 and 1958. This procedure is not completely satisfactory. Deleting the most recent years tends to destroy the predictive value of the model.

In predicting changes in cow numbers two variables have been included in the equations to explain this beef-milk relationship. The beef-milk price ratio lagged one year (N_{6t-1}) is assumed to reflect competition. When the price of beef increases, dairy cow numbers decline. The complementary cyclical relationship between beef and dairy cattle is represented by the variable for the beef cycle. Observations of this variable are the first differences of beef cattle numbers. Transformation of observations to first differences eliminates the effect of the upward trend in beef cattle numbers without destroying the cycles.

Coefficients obtained with milk cow numbers as the dependent variable are shown in Equations 75 and 75A. All signs conform with logic except the sign for the coefficient of the milk-feed price ratio in the latter equation. The negative sign indicates that milk production has declined during periods when the milk-feed price ratio has increased. This negative sign may be due to specification bias.

The coefficient of the beef-milk price ratio is significant at the five percent level in the postwar period. The beef cycle is the most important factor explaining the variance in dairy cow numbers. Thus beef cattle numbers and beef cattle price have an important influence on milk production even in a region where competition in the form of beef cattle production is of minor importance.

The R^2's for equations predicting Northeast cow numbers are significantly higher than those obtained in the Lake

States. However, the possibility of specification bias is not eliminated. Therefore, estimated cow numbers were not substituted for observed numbers in computing the equations for production per cow.

Coefficients for equations with production per cow as the dependent variable are shown in Table 22. The independent variables for these equation include trend (N_{4t}) and (N'_{4t}), the milk-feed price ratio in the current and in the previous year (N_{5t} and N_{5t-1}), and cow numbers (N_{2t}). A variable to explain changes in forage production also has been included. This variable is defined as follows:

> N_{8t} = yield per acre of hay in the current year as a percentage of the trend in yield. Observations are for the yield per acre of hay in the Northeast divided by the computed values of the linear trend in yield per acre of hay.

Forage production has been more stable in the Northeast and the influence of drought on milk production has been less severe. However, this variable was included to determine whether or not forage yields had significantly influenced production per cow. The variable reflects principally the pasture conditions in the current year.

Other than trend the most important variable explaining the variance in production per cow is cow numbers. Coefficients for the milk-feed price ratio and for hay yield per acre were not significant in either time period. The standard errors of these coefficients are appreciably higher in the postwar period. The coefficient for hay yield per acre was

Table 22. Supply response through production per cow, Northeast, 1926-58 and 1947-58: showing regression coefficients, standard errors, and R^2 for regression equations using logarithms of observed values.

Equation Number	Time[a] N_{4t}	Milk-Feed price ratio N_{5t}	Milk-Feed price ratio $N_{5(t-1)}$	Milk cow numbers N_{2t}	Hay yield per acre N_{8t}	R^2
(76) 1926-58	.0001447 (.000005107)	.04905 (.06447)	-.04097 (.05938)	-.51422 (.13363)	.02153 (.01742)	.980
(76A) 1947-58	.007131 (.0011088)	.08598 (.18719)	.08193 (.14611)	-.11588 (.37268)	-.01136 (.05082)	.959

[a]Time entered in Equation 76 as the square of the trend variable: 1926 = 1, 1927 = 4, 1928 = 9 ....

positive in Equation 76 and negative in Equation 76A. This conforms with results obtained in the Lake States. The negative sign in the postwar years suggests that during this period the quantity of forage produced has been less important than the quality.

Results of equations explaining production per cow and cow numbers are combined to obtain the elasticity for milk production. The procedure followed has been shown in detail in Equations 52 through 60 in the previous section. The coefficients for the milk-feed price ratio and the beef-milk price ratio in Table 21 are identified as b_2 and b_3. The coefficients for the milk-feed price ratios in Table 22 are identified as d_2 and d_3. The coefficient for milk cows in the same table is d_4. Using these symbols the elasticity for milk is shown in Equation 77:

$$E_m = d_2 + d_3 + b_2(1-d_4) + b_3(1-d_4) \tag{77}$$

The elasticities for milk production were computed ignoring coefficients with signs that did not conform with logic. The elasticities are shown for the 1926-58 period in Equation 78 and for the 1947-58 period in Equation 78A.

$$E_m = .049 + .161(.486) + .025(.486) = .139 \tag{78}$$

$$E_m = .086 + .082 + .080(.884) = .239 \tag{78A}$$

Little confidence can be placed in the magnitudes of the coefficients shown in these equations. Many of the coefficients used to compute the elasticities had comparatively large standard errors. However, these results indicate that the elasticity of response has increased in the postwar period. This increase could be due either to greater certainty of price expectations or to improved knowledge and technological efficiency.

Summary

The Northeast has been heavily concentrated in dairying. The comparative advantage of dairying over other farm alternatives has enhanced the stability of production in the region. Response to change in price has been difficult to identify. Coefficients with "wrong" signs and large standard errors were obtained frequently in the regression analysis. Efforts to obtain more meaningful results by modifying the models were not completely successful.

Estimates of short-run supply elasticities were not consistent. For example, price elasticities for equations in the 1926-58 and the 1947-58 period are lower than those for equations with omitted observations. However, more confidence can be placed in the estimates with observations for 1957 and 1958 deleted as standard errors of the coefficients are lower. In addition, elasticities in these latter equations conform more closely with estimates obtained using first difference and recursive models.

The short-run elasticity of supply for the 1926-58 period in the Northeast appears to be between .15 and .20. For some equations the estimate of elasticity in the postwar period was lower than for the entire thirty-three years. However, the evidence seems to indicate that the elasticity for the 1947-58 period is higher. Based on estimates from single equation and recursive models an appropriate range is .23 to .28. It appears that the elasticity of supply for the postwar years falls somewhere in this range.

Prices of milk and feed are important in explaining the variance in milk supply. However, the results of regressions with cow numbers as the dependent variable suggest that the beef price is also an important factor. In the single equation models, coefficients for the milk-wage price ratio were significant. This significance apparently is due to multicollinearity. Coefficients for the deflated wage index were not significant.

The attempt to determine the elasticity of response under rising and falling prices was unsuccessful. The coefficients for the milk-feed price ratio were not significant and the results must be considered inconclusive.

Estimates of elasticities for milk from the recursive model compared closely with results using single equations. The beef cycle is the most important variable explaining changes in cow numbers. Cow numbers, in turn, has an important influence on changes in production per cow.

SUPPLY RESPONSE IN CALIFORNIA

California is the final region for which empirical estimates of supply response were obtained. The models used and the presentation of results follow the pattern of the previous sections. Coefficients were computed principally for observations in logarithms for the two time periods 1926-58 and 1947-58. As in the Northeast, estimates were not obtained using the Nerlove distributed lags model. The basic data used in computing the regression equations are presented in Tables 43 and 44 of Appendix A.

Two important problems were encountered in the regression analysis for California. First, growth in California milk production has been more rapid than production growth in the other regions. Many of the time series used to explain production variance reflect this upward trend. As a consequence, multicollinearity is a serious problem. The regression models were modified to reduce the problem of multicollinearity in both the single equation and the recursive models. Secondly, although alternatives to dairying exist in California, these alternatives are too numerous to be effectively explained by the inclusion of variables in the regression equation. In theory any number of variables can be included in the model. However, to obtain meaningful coefficients the number of variables in a single equation should be limited to about six. Inability to accurately define the competing alternatives in the regression model may lead to specification bias. None of the alternatives taken individually has had a significant effect on milk production. However, the aggregate effect of alternatives may be relatively important.

The principal variables used in California regressions other than trend were the prices of milk, feed, and beef. The wage rate has not been used primarily because of the difficulty in interpreting the coefficient (explained in the previous section). A change in wage rates does affect the profitability of production. However, the production response should not be the same in California as in the Northeast. The labor used in dry lots is unionized. The number of cows per milker is fixed by union regulation. Therefore, there is no advantage to be gained by introducing labor saving devices. Under these circumstances an increase in the wage rate would result in a decline in milk production. This labor situation could have a serious long-run effect on production in areas affected by union regulations.

No variable was used to reflect changes in forage production. Coefficients for hay yield per acre were not significant in the Northeast. Forage production has been more stable in California than in any of the regions studied. Quality of hay is also higher and more uniform since most of the hay is raised on irrigated land. Therefore, the hypothesis set forth in Appendix D with respect to farm roughage feeding programs under varying weather conditions may have little relevance in California.

Conventional Single Equations

Tables 23 through 26 contain coefficients of variables estimated by regressions with milk production as the dependent variable. The variables of each equation are identified by the heading and the numerical designation. (In this case, C represents California.) The equation number and the time period are identified at the left. The coefficients of each equation are accompanied by the standard errors (in parenthesis below). The R^2 indicating the percentage of the variance in milk production explained by the variables in each equation is shown in the column to the right of the coefficients. The sum of the milk price elasticities represents the sum of the absolute values of coefficients of the milk price variables (e.g., the milk-feed price ratio, and the beef-milk price ratio). Coefficients whose signs were contrary to logic are not included in this sum. In Tables 23 and 25, the Durbin-Watson statistic is computed for equations in the 1926-58 period with the lowest error of the estimate.

The variables used in the equations shown in Table 23 are defined as follows:

C_{1t} = the total milk production in California in the current year. This is the dependent variable.

C_{4t} = time: 1926 = 1, 1927 = 2 ...

C_{5t} = the milk-feed price ratio in the current year. The price per cwt. of milk from combined milk and cream marketings in California is divided by the price per cwt. of concentrate rations fed to milk cows in the Western states.

C_{5t-1} = the milk-feed price ratio in the previous year

C_{5t-2} = the milk-feed price ratio two years previous

C_{6t-1} = the beef-milk price ratio in the previous year. The price per cwt. received for beef cattle in the Pacific States is divided by the price per cwt. of milk from combined milk and cream marketings in California.

The price series for feed and beef cattle are regional. Both series were closely correlated with corresponding state prices in the years that these were available.

Equations 79 and 79A contain coefficients for trend, (C_{4t}), and the milk-feed price ratio in the previous year, (C_{5t-1}). Both forms of the exponential function for time used previously (i.e., 1926 = 1, 1927 = 2, etc. and 1926 = 1, 1927= 4, etc.) were tested to determine which form of the variable gave the better explanation of growth in California production. The unsquared values have been used in the equations for both time periods. The milk-feed price ratio in the previous year was introduced in the first set of equations because, as subsequent equations illustrate, the major response to a change in the price apparently occurs after a lag of one year. The coefficient for the lagged milk-feed price ratio is significant in Equation 79. This variable and the trend explain much of the variance in milk production for the 1926-58 period. The elasticity for the milk-feed price ratio in Equation 79A is slightly higher than that for the previous equation. However, the standard error is large and the coefficient is not significant.

Table 23. Milk supply response, California, 1926-58 and 1947-58: showing regression coefficients, standard errors, R^2, the sum of the elasticities for milk, and the Durbin-Watson statistic for regression equations using logarithms of observed values.[a]

Equation Number	Time C_{4t}	Milk-feed price ratio C_{5t}	Milk-feed price ratio $C_{5(t-1)}$	Milk-feed price ratio $C_{5(t-2)}$	Beef-milk price ratio $C_{6(t-1)}$	R^2	Sum of E for milk price	D-W stat. d'
(79) 1926-58	.01058 (.0002563)		.22377 (.07832)			.983	.224	
(79A) 1947-58	.01149 (.001492)		.28394 (.22290)			.917	.284	
(80) 1926-58	.01061 (.0002623)	.06728 (.09647)	.19059 (.09227)			.983	.258	
(80A) 1947-58	.01216 (.001457)		.11356 (.23536)	.29093 (.18907)		.936	.394	
(81) 1926-58	.01062 (.0002662)	.06621 (.09793)	.16867 (.10832)	.03707 (.09224)		.983	.272	
(81A) 1947-58	.01465 (.002386)	-.36511 (.28327)	.14711 (.22769)	.27302 (.18224)		.948	.420	
(82) 1926-58	.01103 (.0003906)	.10760 (.11308)	.11408 (.09329)	.005372 (.10048)	-.06171 (.04334)	.985	.289	.597

[a]Abbreviations for column headings are as follows: E = elasticity, D-W = Durbin-Watson, stat. = statistic.

The milk-feed price ratio in the current year was added in Equation 80. Although the coefficient is not significant, the combined elasticity of the two price ratios (i.e., the sum of the elasticities for milk price) is .258. The elasticity is thus increased by the extension of the time period of adjustment.

It was not possible to obtain a meaningful coefficient in the postwar period for the milk-feed price ratio in the current year (see Equation 81A). The correlation between time and the milk-feed price ratio in the current year is high, resulting in multicollinearity. Therefore, Equation 80A contains coefficients for the milk-feed price ratio lagged one and two years. The standard errors for both coefficients are high. The sum of the elasticities for milk price is .394.

Variables for the milk-feed price ratio in three time periods (t, t-1, and t-2) are included in Equations 81 and 81A. As previously noted, the coefficient for the milk-feed price ratio in the current year is negative in the latter equation. The coefficients in Equation 81 show the time path of adjustment over the three year period. Production response was highest in the second year and declined sharply in the third. The sum of the elasticities for milk price representing the total response over the three year period is .272. A one percent change in the price of milk is predicted to bring about a .272 percent change in the quantity of milk produced by the end of the third year.

Equation 82 contains a variable for the beef-milk price ratio lagged one year. The variable was chosen to explain changes in culling rates due to a change in the beef price. As in the Northeast, no series for slaughter cow prices was available. Therefore, the price of beef cattle was used in the ratio. The 1947-58 model has not been expanded to include the beef-milk price ratio because observations were closely correlated with lagged observations for the milk-feed price ratio, (C_{5t} and C_{5t-1}).

Factors which affect the culling rate are important in California because of the high rate of turnover in cow numbers. Individual cows are retained a comparatively short time in some dairy herds. An average of 50 percent of the herd is replaced each year in the dry lot operations. There are three important factors which influence the rate of turnover: the prices of milk, of beef, and of dairy replacements. The price of replacements is more closely correlated with beef than with milk prices. A drop in production due to a change in cow numbers could be the result of a high rate of culling, a low rate of replacement, or some combination of these factors. Because of the high correlation between the price of beef cattle and of replacement cows, these variables cannot be included in the same equation. Hence, the influence of these two factors cannot be isolated.

In Equation 82 the coefficient for the beef-milk price ratio is negative but not significant. The inclusion of this variable raises the sum of the elasticities for milk price to .289 in the 1926-58 period. The Durbin-Watson d' statistic computed for Equation 82 was .597. The Durbin-Watson test indicates the presence of positive serial correlation in the residuals.

Tintner (57, p. 303) states that multiple regression of a set of variables which are deviations from the linear trend is equivalent to including time explicitly in the regression. However, coefficients may not be equivalent when results are affected by multicollinearity. To reduce the problem of multicollinearity in the postwar series observations of variables were expressed as deviations from the trend.

Deviations from the trend were obtained in the following manner. Regressions were computed to obtain expected values of milk production (C_{1t}), the milk-feed price ratios (C_{5t}, C_{5t-1}, and C_{5t-2}) and the beef-milk price ratio (C_{6t-1}) with time (C_{4t}) as the independent variable. The trend was fitted for the 1926-58 period (1926 = 1, 1927 = 2, etc.). The 1926-58 trend was used to give a more adequate explanation of long-run secular change. Deviations from the trend were obtained for the 1947-58 period by subtracting expected values from observed values. This procedure was followed for each of the five variables.

Regression equations were computed for the 1947-58 period with observations of variables expressed as deviations from the trend. The coefficients obtained with milk production as the dependent variable are shown in Table 24. Equations 83 to 86 are comparable to Equations 79 to 82 of the previous table in terms of the variables used.

Table 24. Milk supply response, California, 1947-58: showing regression coefficients, standard errors, R^2, and the sum of the elasticities for milk price for regression equations using logarithms of observations expressed as deviations from the 1926-58 trend.[a]

Equation no.	M/F p. ratio C_{5t}	M/F p. ratio $C_{5(t-1)}$	M/F p. ratio $C_{5(t-2)}$	B/M p. ratio C	R^2	Sum of E for milk
(83) 1947-58		.31926 (.19755)			.207	.319
(84) 1947-58	.07866 (.18946)	.28082 (.22610)			.222	.360
(85) 1947-58	.13426 (.20415)	.18289 (.25864)	.17426 (.21001)		.284	.491
(86) 1947-58	.09593 (.22265)	.22296 (.27823)	.06470 (.28706)	-.04934 (.08347)	.318	.433

[a]Abbreviations: M/F p.-milk-feed price; B/M p.-beef-milk price; E-elasticity.

The coefficient for the milk-feed price ratio in Equation 83 compares closely with the coefficient for the corresponding equation in Table 23 (Equation 79A). This is to be expected since results are not affected by multicollinearity. In Equations 84 through 86 the sign for the milk-feed price ratio in the current year is positive. The positive sign contrasts with the negative sign obtained for this coefficient in Equation 81A.

The conversion of time series to deviations from the trend appears to have successfully avoided the problem of multicollinearity. All of the signs of coefficients in Table 24 conform with logic. However, the standard errors of the regression coefficients are comparatively large. None of the coefficients are statistically significant at the five percent level. The R^2 indicates that in Equation 86 less than one third of the variance of deviations from the milk production trend has been explained.

The observations of price variables in the equations of Table 25 have been deflated by the index of prices received. The dependent variable, (C_{1t}), and the trend variable, (C_{4t}), used in these equations have been defined previously. Other variables are defined as follows.

C_{9t} = milk price in the current year. The price per cwt. of milk from combined milk and cream marketings in California is divided by the U. S. index of prices received for all farm products.

C_{9t-1} = the milk-feed price ratio in the previous year deflated

C_{10t} = the price per cwt. of concentrate ration fed to milk cows for the Western states divided by the U. S. index of prices received for all farm products.

C_{11t-1} = the price per cwt. of beef in the Pacific Coast states divided by the index of prices received for all farm products.

Equations 87 and 87A contain variables for trend, (C_{4t}), the milk price in the previous year, (C_{9t-1}), and the feed price in the current year, (C_{10t}). The coefficients for trend and the lagged milk price are significant in both time periods.

Table 25. Milk supply response, California, 1926-58 and 1947-58: showing regression coefficients, standard errors, R^2, the sum of the elasticities for milk, and the Durbin-Watson statistic for regression equations using logarithms of observed values.[a]

Equation Number	Time C_{4t}	Milk price deflated C_{9t}	Milk price deflated $C_{9(t-1)}$	Feed price deflated C_{10t}	Beef price deflated $C_{11(t-1)}$	R^2	Sum of E for milk price	D-W stat. d'
(87) 1926-58	.01056 (.0002679)		.17799 (.07393)	-.08935 (.10009)		.982	.178	
(87A) 1947-58	.01107 (.0009335)		.39743 (.10194)	-.02035 (.12312)		.967	.397	
(88) 1926-58	.01063 (.0002736)	.11473 (.09848)	.13543 (.09661)	-.15587 (.11507)		.983	.250	
(88A) 1947-58	.01182 (.002025)	-.10298 (.24344)	.39016 (.10898)	.10191 (.23256)		.968	.287	
(89) 1926-58	.01110 (.0004300)	.16171 (.10233)	.11344 (.08239)	-.16767 (.11339)	-.06917 (.04899)	.984	.275	.610
(89A) 1947-58	.01265 (.001322)	.02878 (.16101)	.17148 (.09598)	.03157 (.15055)	-.18291 (.05506)	.989	.200	

[a]Abbreviations for column headings are as follows: E = elasticity, D-W = Durbin-Watson, stat. = statistic.

The milk price elasticity in the 1947-58 period is more than double that for the 1926-58 period. The coefficients for feed price are negative but not significant. The low magnitude of the feed coefficients suggests that lagging this variable one year might have been more appropriate.

The price of milk in the current year has been added in Equations 88 and 88A. The signs of coefficients in the 1926-58 period conform with logic. However, a negative sign for the milk price in the current year and a positive sign for the feed price suggests multicollinearity. Coefficients for the beef price in Equations 89 and 89A are both negative. The coefficient is larger in the postwar period indicating that farmers have become more responsive to a change in the price of beef.

The use of observations of deflated prices lowered the standard errors of the regression coefficients in the 1947-58 period (compare Tables 23 and 25). For example, the standard errors of coefficients for milk and feed in Equation 87A are approximately one-half of the standard error for the milk-feed price ratio in Equation 79A.

The Durbin-Watson d' statistic computed for Equation 89 is .610. This figure is close to the value obtained for d' in Equation 82 (Table 23). The test indicates positive serial correlation in the residuals. The residuals of Equation 89 when plotted against the years show a definite cyclical fluctuation. Residuals are negative during the depression and early war years, and during the period 1949-52. The positive serial correlation may be due to the omission of relevant variables.

Observations of variables were expressed as first differences of logarithms to determine the effect of this transformation on coefficients and standard errors. The price varia-

Table 26. Milk supply response, California, 1926-58 and 1947-58: showing regression coefficients, standard errors, R^2, and the sum of the elasticities for milk price for regression equations using first differences of logarithms.[a]

Equation no.	Milk p. deflated C_{9t}	Milk p. deflated $C_{9(t-1)}$	Feed p. deflated C_{10t}	Beef p. deflated $C_{11(t-1)}$	R^2	Sum of E for milk
(90) 1926-58	.02430 (.09533)	.13394 (.05581)	-.02943 (.10208)	-.03120 (.04229)	.202	.158
(90A) 1947-58	.07419 (.14818)	.18686 (.08564)	-.06507 (.16517)	-.13679 (.08002)	.693	.261

[a]Abbreviations: p.-price; E-elasticity.

bles used in Equation 90 and 90A are the same as those used in Equations 89 and 89A. All signs of coefficients in Table 26 conform with logic. The magnitudes of the coefficients in the 1926-58 period are generally lower for computations using first differences. For example, the sum of the elasticities for milk is .275 in Equation 89 but only .158 in Equation 90. The low R^2 in this latter equation indicates that only about one-fifth of the year to year variance in milk production has been explained by these variables. This low R^2 is further evidence that relevant variables may have been omitted. The

R^2 in Equation 90A is much higher than that for the previous equation. The lagged milk price and the lagged beef price give the most significant explanation of year to year changes in milk production. The sum of the elasticities for milk in the postwar period is close to double that for the 1926-58 period. The transformation of observations to first differences of logarithms has had very little effect on the magnitudes of the standard errors of coefficients. These equations also indicate that the elasticity of response has increased in the postwar period.

Table 27. Estimate of milk production in California for 1959: showing observed value, estimated values, percent error of the 1959 estimate, and the error of the estimate as a percentage of the mean.

	Milk production million lbs. 1959	Error percent 1959	Error of estimate as percent of mean
Observed value	7,974	-	-
Estimate from Equation 96	7,825	1.87	2.70
Estimate from Equation 96A	7,849	1.57	.84

Equations 96 and 96A were used to obtain predictions of milk production for the year 1959. These were the equations with the lowest error of the estimate for each time period. The observed value, the estimates of milk production, and the errors associated with these estimates are shown in Table 27. Both equations underestimate production. This underestimation was apparently due to the fact that an increase in beef prices in 1958 was not followed by a decline in milk production in the following year. There is, of course, some indication that coefficients in Equation 96A are affected by multicollinearity. The coefficient for feed price, for example, is positive. However, a regression equation remains valid for prediction even when multicollinearity is present.

The Irreversible Model

The single equation analysis provides evidence that the major response to a change in milk price in California occurs in the following year (see, for example, Equations 87 and 88). This lag was considered in the grouping of years according to long and short-run price movements to test the hypothesis of irreversibility. Observations were grouped according to long-run price change following the procedure of the previous sections. A two year lag was allowed for response to a change in the direction of price. In subgrouping into categories of short-run response, the selection of observations was based upon the direction of price movement in the previous year. For example, if the price rose in the year t-1, the observation of milk production in the year t was placed in a category of short-run rising price. (The grouping of years according to the four categories of short and long run price change is shown in Appendix E.)

The coefficients in Table 28 were computed using time (L_{4t}) and the milk-feed price ratio in the previous year

Table 28. Milk supply response under rising and falling prices, California: showing regression coefficients, standard errors, R^2, and the number of observations for regression equations using logarithms of observed values.[a]

Equation no.	Time C_{4t}	M/F p. ratio C_{5t-1}	R^2	No. of observ.
(91)				
Rising long-run	.001039	.18219	.990	17
Rising short-run	(.0002757)	(.10110)		
(91A)				
Rising long-run	.01043	-.09497	.998	4
Falling short-run	(.0005297)	(.21876)		
(92)				
Falling long-run	.01166	.35624	.990	8
Falling short-run	(.0006145)	(.13691)		
(92A)				
Falling long-run	.01145	.14675	.998	4
Rising short-run	(.0009128)	(.19431)		

[a]Abbreviations: M/F p.-milk-feed price; observ.-observations.

(L_{5t-1}) as the independent variables. The elasticities for the milk-feed price ratio are higher when short and long-run price change is in the same direction. For example, coefficients for L_{5t-1} in Equations 91 and 92 are higher than those in Equations 91A and 92A. However, the results indicate that the short-run response under a rising long-run price is lower than the short-run response under a falling long-run price. For example, the milk-feed price elasticity in Equation 91 is only about one-half of that in Equation 92. This apparent contradiction may be due to the omission of relevant variables.

In spite of the high R^2's, only one of the price coefficients is significant. The single equation analysis indicates that a change in the beef price has been important in explaining milk production variance. However, as previously noted it is not possible to consider an additional price variable without further subgrouping which would reduce the number of observations in each category.

The Recursive Model

Estimates of milk supply elasticity were obtained following the procedure for the other regions. Coefficients were estimated using cow numbers and production per cow in turn as the dependent variable. These results were combined to estimate milk production response in the two time periods. A full discussion of the logic of the recursive model and the computational procedure used is given in pages 106 to 108.

The first step in the computation of milk price elasticities is to predict cow numbers. The variables used in the regression equations with cow numbers as the dependent variable are defined as follows:

C_{2t} = the number of cows on farms in California in the current year. This is the dependent variable.

C'_{5t-2} = the milk-feed price ratio lagged two years, variable moving average. The price per cwt. of milk from combined milk and cream marketings in California is divided by the price per cwt. of concentrate ration fed to milk cows in the Western states. Observations are for a variable moving average of this ratio lagged two years.

C_{13t-1} = the beef cycle in the previous year. The first difference of the number of cattle and calves kept for beef in the United States Jan. 1 is lagged one year.

C_{15t-1} = the milk cow-milk price ratio in the previous year. The average price per head received by farmers for milk cows in California is divided by the price per cwt. of milk from combined milk and cream marketings in California.

Table 29. Supply response through cow numbers, California, 1926-58 and 1947-58: showing regression coefficients, standard errors, and R^2 for regression equations using logarithms of observed values.[a]

Equation no.	Time C_{4t}	M/F p. ratio $C'_{5(t-2)}$	Beef cattle cycle $C_{13(t-1)}$	M. cow/M price ratio $C_{15(t-1)}$	R^2
(93) 1926-58	.005929 (.0003404)	.431751 (.12359)	-.14853 (.10073)	-.007716 (.04275)	.958
(93A) 1947-58	.004315 (.0003898)	.39557 (.13158)	.14009 (.06777)	-.07759 (.03579)	.967

[a]Abbreviations: M/F p.-milk-feed price; M. cow/M-milk cow milk price.

Trend was used in equations for both time periods to account for the increase in cow numbers due to population growth. The variable moving average of the milk-feed price ratio was computed to remove short-run fluctuations in the ratio which were assumed not to influence cow numbers. The two remaining variables, the beef cycle, and the milk cow-milk price ratio explain the beef-milk relationship.

The high R^2 values obtained in Equations 93 and 93A are explained principally by the significant coefficients for trend. Coefficients for the milk-feed price ratio are also significant. The magnitude of these coefficients is comparatively high. The coefficient for the beef cattle cycle is negative in the 1926-58 period. The sign implies that when beef cow numbers increase dairy cow numbers decline. Cyclical fluctuations in cattle numbers have not been as large in California as in the other regions studied. Economic growth and the comparative stability in crop and forage production have tended to dampen the cycles. The influence of factors affecting cattle numbers has not been felt with equal force on both sides of the continental divide. The number of beef cattle in California might have been a more appropriate variable to use.

The milk cow-milk price ratio was used to explain changes in cow numbers due to changes in culling and replacement rates. The average price per head received by farmers for milk cows includes sales of slaughter and replacement cows. The coefficients of milk cow-milk price ratio are negative in both time periods, but significant only in the postwar period.

Although the R^2's for the equations in Table 29 are comparatively high, the Durbin-Watson test indicates positive serial correlation in the residuals of Equation 93. Therefore, in the computation of equations for production per cow, actual rather than estimated values of cow numbers were used.

The problem of multicollinearity occurs in the equations predicting production per cow. Observations of variables for time and cow numbers are closely correlated. The model was modified in an attempt to reduce this problem. Observations were transformed to first differences. However, coefficients for the milk-feed price ratio in these equations were negative. Coefficients were obtained with cow numbers only in first differences. (This corresponds to the procedure used for the beef-cycle variable, C_{13t-1}.) The most reasonable estimates in terms of the logic of the signs and the magnitude of the standard errors of coefficients are shown in Table 30. In this table observations of price variables are deflated.

Table 30. Supply response through production per cow, California, 1926-58 and 1947-58: showing regression coefficients, standard errors, and R^2 for regression equations using logarithms of observed values.[a]

Equation no.	Time[b] C_{4t}	Milk p. deflated $C_{9(t-1)}$	Feed p. deflated C_{10t}	Milk cow numbers C_{2t}	R^2
(94) 1926-58	.0001474 (.00001474)	.12504 (.05409)	.06266 (.07181)	-.03853 (.08252)	.961
(94A) 1947-58	.008141 (.001134)	.16481 (.10183)	-.01105 (.08537)	-.08749 (.29019)	.968

[a]Abbreviation: p.-price.

[b]Time entered in Equation 103 as the square of the trend variable: 1926 = 1, 1927 = 4 ...

The independent variables used in Equations 94 and 94A were trend (C_{4t} and C'_{4t}), the milk price in the previous year deflated (C_{9t-1}), the feed price deflated (C_{10t}), and the number of milk cows on farms (C_{2t}). These variables have been defined previously. Squared observations of time were used to indicate the trend in milk production per cow in the 1926-58 period. As in the other regions, the average annual rate of increase in production per cow has been much higher in the postwar period (see Table 1, page 46). The variables for trend and the lagged milk price explain much of the variance in production per cow. Coefficients for the other variables are not significant.

The coefficients for cow numbers are much smaller than those obtained in other regions. There appears to be no logical reason why a change in cow numbers should have little effect on production per cow in California. Coefficients for cow numbers in the Lake States and the Northeast (Tables 14 and 22) compare favorably with regional estimates obtained by Halverson (31) for the 1939-54 period. Halvorson's study included six regions of the United States. He used first differences of observations for the winter and summer feeding periods. The majority of his estimates of the coefficients for cow numbers were between -.40 and -.70. The coefficients for the Western region were -.36 in the summer period and -.49 in the winter period.

The coefficients of the equations for production per cow and cow numbers are combined to obtain the elasticity for milk production according to the following procedure. The coefficients for the milk-feed price ratio and the milk cow-milk price ratio in Table 29 are identified as b2 and b3.

The coefficients for the milk price and cow numbers in Table 30 are identified as d_2 and d_4. The elasticity of milk production (E_m) is shown in Equation 95.

(95) $$E_m = d_2 + b_2(1-d_4) + b_3(1-d_4)$$

Substituting the coefficients for the symbols, the elasticities for the 1926-58 and 1947-58 periods are shown in Equations 96 and 96A respectively.

(96) $$E_m = .125 + .432\ (.971) + .008\ (.931) = .552$$

(96A) $$E_m = .165 + .396\ (.913) + .078\ (.913) = .598$$

These estimates of supply response are considerably higher than those for the single equation analysis. The high coefficients obtained appear to be due principally to a biased coefficient for cow numbers. The nature of this bias has been discussed with reference to the estimates obtained by Halvorson. The elasticities were recomputed substituting an assumed value of -.45 for the coefficient of cow numbers (d_4) in each time period. The elasticities obtained following this procedure were .367 for the 1926-58 period and .427 for the 1947-58 period. These values compare closely with earlier estimates using single equations.

Summary

The analysis of supply response in California has been complicated by the sharp upward trends in many of the price and production series used. Modification of the models has helped to overcome the problem of multicollinearity. Reasonable estimates of elasticities have been obtained. However, serial correlation in the residuals indicates that specification bias may exist.

The short-run elasticity of supply (three year adjustment period) for the 1926-58 period appears to be between .25 and .30. The estimate of elasticity using first differences fell below this range, and the estimate from the recursive model was above the range. There is evidence from both the single equation and recursive models that the elasticity in the postwar years has increased. The elasticity for the 1947-58 period appears to be between .40 and .45.

The prices of milk, feed, and beef cattle are all important in explaining variance in milk production. Farmers appear to have become more responsive to a change in the beef price in the postwar period.

The results of the irreversible model are contradictory. Elasticities under long-run falling price exceed elasticities under long-run rising price. At the same time elasticities are higher under short-run price rise than under a short-run price fall. This unsatisfactory explanation of production response under rising and falling prices may be due to the omission of relevant variables.

Estimates of elasticities using the recursive model were comparatively high. These high coefficients are due to (1) high coefficients for the lagged milk-feed price ratio in the prediction of cow numbers and (2) low coefficients for cow numbers in the prediction of production per cow. The low coefficient for cow numbers may be the result of multicollinearity. The estimates of milk supply using an assumed value of -.45 for the milk cow coefficient appeared more accurate in terms of previous estimates.

REGIONAL COMPARISON OF SUPPLY RESPONSE

Estimates of supply response obtained using regression analysis in the previous sections are collated in this section. Elasticities of milk price coefficients are compared to determine whether the hypotheses set forth in this study should be accepted or rejected. Tables show the elasticities of supply computed by summing the coefficients of variables containing the price of milk. Prior to making these comparisons supply response in the United States is analyzed briefly. This analysis permits a comparison of the coefficients for the United States with those for each of the regions studied.

Supply Response in the United States

Least squares regressions were computed for the United States first with observations of price variables expressed as ratios, and then with prices deflated. Coefficients from these equations are shown in Tables 31 and 32. These tables are organized following the procedure of the previous sections. In the numerical designation of variables the symbol U stands for United States.

The variables used in the regressions for the United States are defined as follows:

U_{1t} = the total milk production in the United States in the current year

U_{4t} = time: 1926 = 1, 1927 = 2 ...

U_{5t} = the milk-feed price ratio in the current year. The price per cwt. of milk from combined milk and cream marketings in the U. S. is divided by the price per cwt. of concentrate ration fed to milk cows in the U. S.

U_{5t-1} = the milk-feed price ratio in the previous year

U_{6t-1} = the beef-milk price ratio in the previous year. The price per cwt. for canner and cutter cows in Chicago is divided by the price per cwt. of milk from combined milk and cream marketings in the U. S.

U'_{7t-2} = the hog-milk price ratio two years previous. A variable moving average of the price per cwt. packer and shipper purchases in Chicago is divided by the price per cwt. of milk from combined milk and cream marketings in the U. S.

U_{9t-1} = the milk price in the current year deflated. The price per cwt. of milk from combined milk and cream marketings in the U. S. is divided by the U. S. index of prices received for all farm products.

U_{17t-1} = the meat price index in the previous year deflated. The U. S. index of prices received by farmers for meat animals is divided by the U. S. index of prices received by farmers for all farm products.

U_{10t} = the feed price in the current year deflated. The price per cwt. of concentrate fed to dairy cows in the U. S. is divided by the U. S. index of prices received for all farm products.

The emphasis in the selection of variables has been placed upon the prices of competing livestock products. Throughout the Corn Belt and in many other sections of the country competition between dairying and other forms of livestock is strong. Many of the physical inputs for various livestock enterprises are similar. Competition does exist between dairying and some crop enterprises, but this competi-

Table 31. Milk supply response, United States, 1926-58 and 1947-58: showing regression coefficients, standard errors, R^2, and the sum of the elasticities for milk for regression equations using logarithms of observed values.[a]

Equation Number	Time U_{4t}	Milk-feed price ratio U_{5t}	Milk-feed price ratio $U_{5(t-1)}$	Beef-milk price ratio $U_{6(t-1)}$	Hog-milk price ratio $U_{7'(t-2)}$	R^2	Sum of E for milk price
(97)	.003806	.18620	.16956	-.02275	.07859	.935	.379
1926-58	(.0007458)	(.07018)	(.06778)	(.02271)	(.04550)		
(97A)	.002300	.07417	.18728	-.07224	-.02424	.915	.357
1947-58	(.001435)	(.16324)	(.12912)	(.02773)	(.07225)		

[a]Abbreviation for column heading is as follows: E = elasticity.

Table 32. Milk supply response, United States, 1926-58 and 1947-58: showing regression coefficients, standard errors, R^2, and the sum of the elasticities for milk for regression equations using logarithms of observed values.[a]

Equation Number	Time U_{4t}	Milk price deflated U_{9t}	Milk price deflated $U_{9(t-1)}$	Feed price deflated U_{10t}	Index of meat prices deflated $U_{17(t-1)}$	R^2	Sum of E for milk price
(98)	.003802	.11029	.17876	-.21952	-.02772	.931	.289
1926-58	(.0002181)	(.08347)	(.08164)	(.08096)	(.03204)		
(98A)	.002468		.26232	-.07766	-.13482	.968	.262
1947-58	(.0004619)		(.05621)	(.07960)	(.05577)		

[a]Abbreviation for column heading is as follows: E = elasticity.

tion is somewhat more localized. No single crop has an important effect on the aggregate figures.

Table 31 contains coefficients computed from regression equations with the following variables: time (U_{4t}), the milk-feed price ratio in the current and previous year (U_{5t} and U_{5t-1}), the beef-milk price ratio in the previous year (U_{6t-1}) and the hog-milk price ratio two years previous (U'_{7t-2}). The hog and beef prices used are the same as those used in the Lake States analysis. Chicago is centrally located and the largest livestock market. Chicago prices were assumed to reflect price changes for hogs and beef cattle throughout the United States.

Equation 97 was computed for the 1926-58 period. Variance in milk production is explained principally by trend and the two milk-feed price ratios. The coefficient for the beef-milk price ratio is negative but not significant. The coefficient for the hog-milk price ratio is positive. This sign does not conform with logic. An increase in the hog price is predicted to bring an increase in milk production. The "wrong" sign for the hog price coefficient appeared earlier in the analysis for the Lake States. Competition from hogs may have been adequately explained by the inclusion of the feed grain price.

The coefficients for the milk-feed price ratio were positive, but not significant in Equation 97A. An important part of the variance in production is explained by changes in the beef-milk price ratio. The hog-milk price ratio is negative but not significant. The sums of the elasticities for milk price in the two time periods compare very closely and are both slightly above .35.

Observations of price were deflated in computing the equation shown in Table 32. The price series used were the same as for the previous table with one exception. The U. S. index of meat prices was substituted for the Chicago prices of beef and hogs. This index is heavily weighted by hog and beef prices (approximately 85 percent). There are two advantages to using the index of meat animals, (1) the index is based upon national rather than local prices, and (2) combining hog and cattle prices conserves one degree of freedom.

The variance in Equation 98 is again explained principally by trend and the prices of milk and feed. The index of meat prices in the 1926-58 period is negative but not significant. The milk-feed price ratio in the current year was omitted from Equation 98A to avoid multicollinearity. The coefficients for lagged milk and meat prices are both significant. The high negative elasticity for the meat index provides evidence that competition between dairying and other forms of livestock enterprise has increased in the postwar period. This has not been true in all regions of the United States. For example, the rise in meat prices relative to dairy prices has weakened the competitive position for dairying in parts of the Corn Belt. At the same time, the competitive position of hogs and beef has been strengthened in much of the Lake States.

Although results are not presented formally, a reversible model was computed for the United States using trend and the milk-feed price ratio as independent variables. Coefficients obtained under a long-run rising price ratio were reasonable. Under a long-run falling ratio, however, the independent variables were closely correlated and coefficients were apparently affected by multicollinearity. Under rising short and long-run milk prices the elasticity for the milk-feed price ratio was .395. For years of long-run rising but short run falling price the elasticity was .166.

The elasticities of supply for milk obtained from Equations 97 to 98A can be compared with the results of other research workers. Cromarty (12) obtained an elasticity for milk price of .212 for the 1929-53 period. Halvorson's (30) estimates ranged from .128 to .185 for the 1929-53 period and from .180 to .362 for the 1941-57 period (depending on the model used). In general, these estimates are below those obtained in this study. This difference is due in part to the fact that variables for both the current and the previous year's milk price have been included in the regression equations in Tables 31 and 32. Thus the time period allowed for adjustment is longer (i.e., definitions of "short-run" differ). The difference in time periods of analysis between studies will also affect the results.

Comparison of Elasticities for Single Equations

Short-run estimates of supply response computed from single equation regression models are compared in this section. Emphasis is placed upon production response due to changes in the price of milk. Comparisons are made between the coefficients representing the "sums of the elasticities for milk price." These coefficients have been computed for each equation and are shown in the column to the right of the R^2's in the tables of the previous sections. Comparisons have been made according to the following procedure.

Equations were divided into three categories based upon the manner in which observations of price variables were expressed in the equation. Observations of variables were expressed (1) as ratios, (2) as deflated prices, and (3) as first differences. These three categories are represented in Tables 33, 34 and 35. The sums of the elasticities for milk price shown in each of these tables are principally for the "fully expanded" models in each region. (Exceptions were made only when coefficients appeared to have been affected by multicollinearity.) Equations in the previous sections were expanded to include five or six variables. These expanded models were assumed to give a fairly accurate explanation of production response over a three year adjustment period.

The procedure is illustrated with reference to Table 33 as follows. This table shows the combined elasticities for the milk price obtained when observations were expressed as ratios (e.g., the milk-feed price ratio). The column headings designate the region. The table number from which the coefficients were obtained is shown in parantheses below the heading for each region. The column to the left indicates the time period. The standard error is shown directly below each coefficient in parantheses.

Table 33. Short-run elasticities of milk supply with respect to milk price for the Lake States, the Northeast, California, and the United States, 1926-58 and 1947-58: showing the sum of the elasticities for milk price and standard errors of the sum for regression equations with observations of prices expressed as ratios.[a]

Time period	Lake States (Table 4)	Northeast (Table 16)	California (Table 23)	United States (Table 31)
1926-58	.357 (.050)	.195 (.064)	.289 (.124)	.379 (.086)
1947-58	.368 (.062)	.309 (.225)	.394 (.222)	.357 (.172)

[a]Northeast regressions are for the 1926-58 and 1947-56 periods.

The elasticity for the Lake States in the 1926-58 period is .357. This figure appears in Table 4 in the column entitled "the sum of the elasticities for milk price." The sum has been obtained by adding the absolute values of the coefficients in Equation 27 for the milk-feed price ratios (L_{5t}, L_{5t-1}, and L_{5t-2}) and the beef milk price ratio (L_{6t-1}). (The hog-milk price ratio was omitted from the sum because the sign did not conform with logic.) Equation 27 is the "fully expanded" equation in Table 4. The value .357 thus indicates the short-run elasticity of response. A one percent change in the price of milk is predicted to bring about a .357 percent change in milk production by the end of the third year.

The standard error of the coefficient combines the errors of the coefficients composing the sum of the elasticities for milk price. The standard error is computed according to the following formula:

$$(99) \quad S_{b_1+b_2\ldots b_n} = \sqrt{S^2_{b_1} + S^2_{b_2} \ldots S^2_{b_n} + 2S_{b_1b_2}\ldots 2S_{b_{n-1}b_n}}$$

where:

$S_{b_1-b_2\ldots b_n}$ = the pooled regression standard error

$S^2_{b_i}$ = the variance for b_1

$S_{b_ib_j}$ = the covariance for b_1b_2

The covariance $S_{b_ib_j}$ is obtained from the inverted matrix of the regression solution. The variance and covariance terms of the regression coefficients are added. The square root of this total is multiplied by the standard error of the estimate for the equation. For example, in Equation 27, Table 4 the square root of the sum of the variance and covariance terms for the variables L_{5t}, L_{5t-1}, and L_{6t-1} is 5.738. This sum multiplied by the standard error of the estimate (.00864) gives .050, the standard error for the 1926-58 Lake States coefficient.

An important consideration is whether elasticities are actually different or whether observed differences might have appeared by chance. The coefficients of independent variables

Table 34. Short-run elasticities of milk supply with respect to milk price for the Lake States, the Northeast, California, and the United States, 1926-58 and 1947-58: showing the sum of the elasticities for milk price and standard errors of the sum for regression equations with observations of prices deflated.[a]

Time period	Lake States (Table 7)	Northeast (Table 17)	California (Table 25)	United States (Table 32)
1926-58	.352 (.045)	.171 (.056)	.275 (.096)	.289 (.094)
1947-58	.292 (.049)	.319 (.146)	.397 (.102)	.262 (.056)

[a]Northeast regressions are for the 1926-58 and 1947-56 periods.

Table 35. Short-run elasticities of milk supply with respect to milk price for the Lake States, the Northeast, and California, 1926-58 and 1947-58: showing the sum of the elasticities for milk price and standard errors of the sum for regression equations with observations expressed as first differences.

Time period	Lake States (Table 15)	Northeast (Table 18)	California (Table 26)
1926-58	.287 (.063)	.205 (.054)	.158 (.109)
1947-58	.408 (.058)	.299 (.223)	.261 (.156)

obtained from separate regression equations can be tested to determine whether observed differences are statistically significant. A t test is normally used. The pooled estimate of the common variance can be obtained using information from both equations. The common variance can in turn be used to compute the standard error of the difference between two coefficients. The t value is calculated by dividing the difference between the coefficients (b_1-b_2) by the standard error. The above test is made under the assumption that the population standard errors of the estimates for the two equations are equal.

A less sophisticated procedure has been used to compare coefficients in this section. A confidence interval was established for the elasticity. Upper and lower bounds were computed taking into account the standard errors on which the coefficients are based. Limits were set one standard error above and below the elasticity. Comparison of the confidence intervals gives a good indication as to whether coefficients are significantly different. The limits for the Lake States coefficient in the 1926-58 period are .307 and .407 . The limits for the corresponding Northeast coefficient are .131 and .259. (Northeast coefficients in Tables 33 and 34 are based upon regressions for the 1926-56 and 1947-56 periods as these estimates appeared to be more reliable.) The confidence intervals do not overlap. It is thus assumed that differences between estimates for the Lake States and for the Northeast did not occur by chance. Following this procedure it can be shown that the coefficients for the Northeast and the United States in Table 33 are also significantly different. The lower limit for the confidence interval in the United States is .293.

Confidence intervals for coefficients overlap in almost all other cases. Thus few conclusions can be reached regarding differences in coefficients (whether between regions or between time periods) based upon statistical tests. Comparisons can be made of point estimates of elasticities. When differences between point estimates are large and consistent, this is assumed to be sufficient evidence that differences did not occur by chance. Tables 33, 34, and 35 (showing estimates for different forms of the variable) indicate the consistency of the relationship between coefficients.

Comparisons are made for the 1926-58 period first. It has already been indicated that elasticities are significantly lower in the Northeast than in the Lake States or the United States for the 1926-58 period. The estimates of elasticities for California (Table 33) are between those for the Lake States and the United States on the one hand and those for the Northeast on the other. For example, estimates in Equation 33 are .379 for the United States, .357 for the Lake States, .289 for California, and .195 for the Northeast. This same pattern exists for Table 34 (where coefficients are based on deflated prices) with the exception that the elasticity for the United States is lower than for the Lake States. Coefficients of variables with observations in first differences are generally lower than coefficients in the preceding tables. (Regressions using first differences were not computed for the United States.) The California elasticity in Table 35 (.158) is lower than the estimate for the Northeast (.205).

In summary for the 1926-58 period, there appears to be no significant difference between short-run elasticities for the United States and the Lake States. Elasticities for the other two regions are lower. There is some indication that the elasticity for the Northeast is lower than that for California although first difference analysis does not support this conclusion.

Comparisons are made next for the postwar period. The relationship between elasticities is less consistent. This inconsistency may be due to the fact that standard errors are generally much larger in the 1947-58 period than in the 1926-58 period. There is no apparent difference in the elasticities for the United States and for the Lake States. In both cases estimates are somewhat lower when observations of prices are deflated (see Table 34). California estimates are higher than those for other regions except in the first difference analysis. Estimates of response for the Northeast are appreciably lower than those for the other regions in Tables 33 and 34. However, in the first difference analysis the coefficient for the Northeast is higher than that for California and lower than that for the Lake States.

In brief, less confidence can be placed in statements about differences in elasticities in the postwar period. The inconsistencies encountered may be due in part to the small number of observations in the equations. There is an indication that the elasticity for California is higher than that for other regions. It appears also that the Northeast elasticity is not greatly different from the elasticity for the United States and the Lake States.

Finally, comparisons are made between elasticities in the two time periods. There is little difference in Tables 33 and 34 between the estimates of elasticities for the 1926-58 and 1947-58 periods in the Lake States. Estimates based upon first differences (Table 35) however, indicate a higher elasticity in the postwar period. In the Northeast and California estimates for the postwar period are consistently higher. In both regions these higher estimates are approximately two-thirds again as large as those for the 1926-58 period.

The results of regressions using the recursive model are compared in the following subsection prior to drawing conclusions with respect to the hypotheses tested in this study.

Comparison of Results for the Recursive System

Elasticities for milk in the recursive model are based upon the solution of equations using cow numbers and production per cow respectively as the dependent variables. Therefore, the elasticities computed for these equations are compared first. Elasticities are shown in Tables 36 and 37. These tables are organized according to the procedure used in the three previous tables. As before, the coefficients represent the "sum of the elasticities for milk price." The standard errors are shown in parantheses below the coefficients. The tables from which the elasticities were obtained are shown below the regional headings in parantheses. The equations from which coefficients were obtained are easily identified since only one equation is shown for each time period.

Table 36. Short-run elasticities for cow numbers with respect to milk price for the Lake States, the Northeast, and California, 1926-58 and 1947-58: showing the sum of the elasticities for milk price and standard error of the sum for regression equations with cow numbers as the dependent variable.

Time period	Lake States (Table 13)	Northeast (Table 21)	California (Table 29)
1926-58	.249 (.102)	.186 (.057)	.439 (.105)
1947-58	.272 (.050)	.080 (.017)	.464 (.107)

Table 37. Short-run elasticities for production per cow with respect to milk price for the Lake States, the Northeast, and California, 1926-58 and 1947-58: showing the sum of the elasticities for milk price and standard error of the sum for regression equations with production per cow as the dependent variable.

Time period	Lake States (Table 14)	Northeast (Table 22)	California (Table 30)
1926-58	.159 (.048)	.049 (.065)	.125 (.055)
1947-58	.332 (.086)	.166 (.071)	.165 (.102)

The comparatively low R^2's obtained in equations with cow numbers as the dependent variable suggests that specification bias in these models may be serious. Bias may exist irrespective of the fact that low standard errors indicate that all coefficients are significant. Comparisons are made with the realization that this bias may invalidate the conclusions reached.

The comparison of confidence limits in Table 36 shows that the elasticities for California are significantly higher in both time periods than those for other regions. These high elasticities indicate that short-run changes in milk production through a change in the number of cows has been more important in this than in the other regions. In support of this finding, it has been noted previously that California farmers particularily in the dry lot areas are responsive to changes in beef and milk prices. The rate of turnover in cows in some California herds is extremely high. This rate is greatly influenced in the short-run by changes in price relationships.

Differences in elasticities between time periods are not large in either the Lake States or California. The Northeast elasticity is smaller in the 1947-58 period than in the 1926-58 period. Hence, increase in milk price elasticities since the war has apparently not been the result of an increase in response through cow numbers.

In contrast, there is an appreciable difference between the coefficients in the two time periods in Table 37 for production per cow. Comparing confidence intervals based upon the standard error, the difference in elasticities between time periods is significant in the Lake States (i.e., confidence limits do not overlap). These results indicate that an increase in the elasticity with respect to milk price in the postwar period is due to an increased response through production per cow. Short-run changes in production per cow are achieved principally by a change in the feeding rate. Farmers are apparently more responsive to changes in the milk-feed price relationship.

Table 38. Short-run elasticity of milk supply with respect to milk price for the Lake States, the Northeast and California, 1926-58 and 1947-58: showing the elasticities for milk price computed for the recursive model.

Time period	Lake States	Northeast	California[a]
1926-58	.316	.139	.367
1947-58	.464	-.239	.427

[a]Results obtained using assumed value of -.450 for cow numbers in Equations 94 and 94A.

The elasticities for milk obtained from the recursive models are shown in Table 38. These elasticities were obtained according to the procedure set forth in Equations 52 to 60 (p. 113-115). Computations for the Northeast and California are shown on pages 145 and 168 respectively. The arbitrary value of -.45 was assumed for the coefficients for cow numbers in computing the elasticities for California in Table 38 (see p. 168).

Based upon the recursive models the elasticities with respect to milk price in all regions are higher in the 1947-58 period than in the 1926-58 period. The high postwar elasticity for the Lake States compares with results for the single equations using first differences. There is little difference between the estimates for the Lake States and California. However, coefficients for the Northeast are considerably lower in both time periods than those for the other regions.

In summary, less confidence can be placed in elasticities obtained through the recursive solution. Multicollinearity and inadequate model specification may have seriously biased coefficients. Nevertheless, results in general compare favorably with earlier findings using single equation models. Useful information is also provided through the components of the model equations for cow numbers and production per cow. There is strong indication from these equations that the postwar increase in supply elasticity has been achieved principally through changes in feeding rates.

Conclusions with Respect to Hypotheses

The hypotheses to be tested in this study were set forth in the objectives and were elaborated in a subsequent section following the descriptive analysis of the regions. The conclusions reached are based principally upon the comparison of the results in the previous pages of this section.

The hypothesis that the short-run elasticity of supply is higher for the United States than for the three regions must be rejected. There is no apparent difference between elasticities with respect to milk price for the United States and the Lake States. However, elasticities are appreciably higher for the United States than for the Northeast. This fact suggests that there are regions outside of the major dairy regions represented in this study where elasticities for milk price are higher than those for the aggregate United States. Furthermore, since each estimate represents an "average" for the region and for the time period, there are geographical areas within the regions studied where the response to price is higher.

The hypothesis that the elasticity of supply for the Lake States exceeds the elasticity of supply in other two regions can be accepted for the 1926-58 period but not for the postwar years. In the 1947-58 period the elasticity for California is equal to if not higher than that for the Lake States. Elasticities in the postwar period have apparently increased to a greater degree in the fluid milk sheds.

The evidence suggests that the elasticities for California exceed those for the Northeast in both time periods. The only contradiction to this conclusion is found in the first difference analysis. The results of other single equation models and of the recursive model support this hypothesis.

The hypothesis that elasticities have increased in the fluid sheds in the postwar period is accepted. However, the reason for this rise is not clear. The first difference equations and the recursive models suggest that a rise occurred also in the Lake States. Thus the higher elasticities may not be due entirely to the fact that fluid producers have experienced greater certainty of price expectations. Greater technological efficiency and extended knowledge may have contributed to this higher elasticity of response. For example, farmers apparently have a better understanding of the concept of profit maximization. Although they may not operate at the point where marginal cost equals marginal return, they show greater response to a change in the milk-feed price relationship.

The attempt to test the hypothesis that the elasticity with respect to milk price is higher under rising than under falling prices was not completely successful. Dependable estimates were obtained only for the Lake States. The results for this region show that the elasticity is affected by both long and short-run price changes. The highest elasticity was obtained under rising long and short-run prices. The elasticity was lowest when long-run prices were falling but short-run prices were rising.

SUMMARY AND CONCLUSIONS

This study has been undertaken to extend present knowledge of supply response in dairying. Past research in supply, with but few exceptions, has focused either on the individual farm unit or on the United States as a whole. Farm management specialists have used budgeting and linear programming in the analysis of individual farm units. Price analysts have relied on aggregate time series in conducting regression analysis for the United States. The present study attempts to bridge the gap between these extremes by providing _regional_ estimates of supply elasticities.

Time series regression analysis is employed to complement the linear programming work now under way in the Lake States. This latter study will provide regional supply estimates through the aggregation of results for homogeneous farm groups. The programming model has been constructed with an assumed adjustment period of five years. The short-run predictive results using time series should underestimate the true response, while normative programming analysis is apt to overestimate this response.

Regional analysis has been extended to include the Northeast and California. These together with the Lake States supply over one-half of the nation's milk. A less extensive investigation was made of the elasticity of supply for the United States. The hypotheses tested concern differences in elasticities as related to, (1) different geographical regions, (2) different time periods, and (3) different economic conditions. These hypotheses are based upon a descriptive analysis of the regions studied and upon the conjectures of other economists. Analysis was conducted for two time periods, 1926-58 and 1947-58 under the assumption that prewar and postwar elasticities differed. The hypothesis that the elasticity of supply is higher under rising than under falling prices was tested by grouping years according to milk-feed price movements in the long and short run.

Models were selected to provide as much information as possible on short-run response to price. Variables used in addition to trend were primarily prices of competing products and costs of inputs. These varied according to the region analyzed. In many instances equations were modified to overcome statistical problems and provide meaningful structural coefficients.

Although there is theoretical justification for a simultaneous equation model in dairy supply, past empirical results using simultaneous models have been largely unsuccessful. Therefore, this approach was not used. Emphasis was placed instead upon single equation models, although elasticities also were computed for a two equation recursive system. In the single equation analysis attention was given to the problem of distributed lags and the question of reversibility. The dependent variables of the recursive system were the components of milk production - cow numbers and production per cow. The various models complement one another in providing a more concrete estimate and explanation of short-run supply response for milk.

The analysis for the Lake States indicates that the average short-run elasticity of supply with respect to milk price is between .30 and .35 for both time periods. This short-run elasticity assumes an adjustment period of from two to three years. Farm response to a change in feed price has been stronger than in the other two regions. This more elastic response suggests that changes in feed price may reflect changes in prices of competing livestock products. (Higher meat prices mean higher feed prices.) The fact that the elasticities for competing livestock products are not significant must be considered in this light. Changes in production per cow and cow numbers contribute about equally in determining the short-run elasticity. Finally, there is strong evidence that the supply function for milk is irreversible. The elasticity of supply is highest when short and long-run prices are both increasing, and lowest when short-run prices increase but long-run prices fall.

The lower elasticity of response in the Northeast is due largely to the fact that dairy farming in this region has enjoyed a strong competitive advantage. The short-run elasticity for the 1926-58 period appears to be between .15 and .20. However, the elasticity for the postwar period may have risen by more than 50 percent. The prices of milk and feed are important in explaining variance in milk supply. The results of regression analysis with milk cow numbers as the dependent variable suggest that beef prices are an important factor in postwar supply response. Competition from non-farm alternatives may not have been adequately explained by the variables used.

The short-run elasticity of supply for California is thought to be between .25 and .30 for the 1926-58 period. The elasticity for the postwar period is considerably larger. Some of the estimates for the 1947-58 period exceed .40. This high elasticity may be due in part to the fact that California has experienced a rapid rate of growth. This rapid growth has undoubtedly influenced farmer expectations. The milk-beef price relationship appears to have been even more important than the milk-feed price relationship in explaining short-run supply response in the postwar period. The recursive analysis shows that response through change in cow numbers has been relatively more important than response through production per cow.

Elasticities for the United States are comparable with those for the Lake States. United States estimates represent an "average" of all regions. The fact that elasticities for the United States are comparatively high suggests that there are regions outside of the ones studied where farm response to a change in price is large.

A regional comparison of supply elasticities with respect to milk price has been made in the previous section. From this comparison it can be concluded that elasticities are highest: (1) where competition from livestock alternatives is strong, (2) where market regulation has increased the certainty of price expectations, (3) where the level of economic understanding and technological efficiency is high, and (4) under a situation of rising prices and favorable price expectations. When these factors occur in combination the short-run elasticity may exceed .50. Conversely, when none of these factors are present, the short-run elasticity should approach zero.

This study of regional supply response cannot be considered exhaustive. There are many instances where further investigation through the use of different variables and different forms of the relationship might provide additional information on short-run response. It is also apparent that certain factors which affect supply response cannot be properly incorporated in the regression model. This study is an initial step toward a better understanding of the regional supply response for milk.

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APPENDIX A

The basic data used in this study are presented in the tables of this appendix. For the regression procedures used, it is assumed that observations of the independent variables are made without error. Errors normally do exist, however. The magnitude of these errors is difficult to assess. Nerlove (50, p. 87) in discussing the limitations of time series data lists four sources of error: (1) errors of a conceptual nature, (2) incomplete available data, (3) voluntary misrepresentation, and (4) inadvertent misrepresentation.

Errors in the first two categories are closely associated. Errors of this sort may be the result of using estimates for regions which did not conform exactly with the regions defined in this study. Questions of accuracy may also arise because of differences in the product, and hence, differences in the source and reliability of estimates. Compare, for example, the problem of obtaining price estimates for fluid milk and purchased feed grain in the Northeast with that of obtaining price estimates for manufactured milk and home-grown grain in the Lake States. One would suspect that the former figure might be more readily obtained and more accurate. A third conceptual problem encountered is that of adjusting prices to allow for wartime production payments.

Voluntary misrepresentation may arise wither at the point of collection or at the point of tabulation of data. Farmers and business men may give inaccurate answers intentionally. Statisticians who compile this data may be under some pressure to misrepresent the figures.

The magnitude of error due to sampling procedures can be assessed. However, a more serious source of involuntary error arises from failure of the respondents either to understand the question or to recall information correctly. Mistakes may also occur in the tabulation of the data. Finally, there is some danger that research workers may misunderstand the data or may use it incorrectly.

In brief, the time series used may not accurately represent the production or price values which are desired. Errors may arise from several sources and may affect only one or two observations or a whole series. Errors in observation take on greater significance when production response is comparatively small. The presence of these errors enhances the difficulty of obtaining accurate estimates of coefficients.

Table 39. Basic data used in regression equations of milk supply response for the Lake States.[a]

Year	1 Tot. milk prod. on farms (mil. lb.)	2 Number of milk cows on farms (thous.)	3 Prod. per cow (lb.)	4 Milk price from comb. milk & cream marketing (dollars/cwt.)	5 Concentrate price ave. E.N. Central & W.N. Central (dollars/cwt.)
1924	20,898	4,211	4,963	1.78	1.66
1925	21,543	4,234	5,088	1.86	1.68
1926	21,757	4,186	5,198	1.88	1.42
1927	21,655	4,116	5,261	2.04	1.55
1928	21,644	4,051	5,343	2.08	1.75
1929	22,558	4,130	5,462	2.03	1.64
1930	22,811	4,275	5,336	1.66	1.42
1931	23,249	4,425	5,254	1.22	.96
1932	23,153	4,555	5,083	.94	.71
1933	23,314	4,709	4,951	.97	.81
1934	22,365	4,710	4,748	1.13	1.23
1935	22,562	4,520	4,992	1.33	1.36
1936	23,808	4,519	5,268	1.51	1.34
1937	23,494	4,536	5,179	1.59	1.62
1938	24,597	4,561	5,393	1.32	1.03
1939	24,895	4,598	5,414	1.23	1.05
1940	26,019	4,705	5,530	1.38	1.17
1941	27,573	4,830	5,709	1.72	1.31
1942	28,530	4,992	5,715	2.03	1.65
1943	28,092	5,072	5,539	2.78[b]	2.06
1944	27,672	5,075	5,453	3.11[b]	2.35
1945	29,044	5,005	5,803	3.39[b]	2.29
1946	29,185	4,914	5,939	3.92[b]	2.71
1947	28,873	4,763	6,062	3.58	3.35
1948	27,422	4,522	6,064	4.11	3.51
1949	28,653	4,427	6,472	3.10	2.62
1950	28,283	4,372	6,469	3.10	2.76
1951	28,289	4,318	6,551	3.75	3.21
1952	28,756	4,354	6,604	3.97	3.31
1953	30,043	4,479	6,708	3.50	3.02
1954	30,311	4,519	6,707	3.19	2.95
1955	30,764	4,497	6,841	3.20	2.74
1956	31,747	4,489	7,078	3.36	2.65
1957	32,568	4,450	7,356	3.39	2.60
1958	33,268	4,327	7,693	3.27	2.46

[a]Abbreviations: tot.-total; prod.-production; comb.-combined; ave.-average; mil.-million; thous.-thousand.

[b]Includes an allowance for dairy production payments Oct. 1, 1943 through June 30, 1946.

Table 40. Basic data used in regression equations of milk supply response for the Lake States[a]

Year	1 Price cutter & canner cows Chicago (dollars/cwt.)	2 Price all packer and shipper hogs Chicago (dollars/cwt.)	3 Yield per acre hay (tons)	4 Number of cattle & calves on farm Jan. 1 in U.S. (thous.)	5 Index of prices rec'd. by farmers 1910-14=100 (index)
1924	3.08	8.11	1.35	35,121	160
1925	3.48	11.81	1.20	32,315	164
1926	4.25	12.34	1.11	29,720	160
1927	5.11	9.95	1.51	27,378	159
1928	6.49	9.22	1.28	26,232	162
1929	6.32	10.16	1.40	26,975	160
1930	4.49	9.47	1.18	27,921	151
1931	2.96	6.16	1.05	29,059	130
1932	2.07	3.83	1.23	30,436	112
1933	2.01	3.94	1.09	33,420	108
1934	2.05	4.65	.79	36,381	120
1935	3.78	9.27	1.58	32,489	124
1936	4.04	9.89	1.13	32,395	124
1937	4.54	10.02	1.41	31,245	131
1938	4.63	8.09	1.54	30,475	124
1939	4.84	6.57	1.40	30,403	123
1940	4.65	5.71	1.55	31,877	124
1941	5.68	9.45	1.52	34,372	133
1942	7.22	13.70	1.71	37,188	152
1943	8.18	14.31	1.62	40,964	171[b]
1944	7.32	13.57	1.49	44,077	182[b]
1945	8.30	14.66	1.64	44,724	190[b]
1946	9.12	18.40	1.44	43,686	208[b]
1947	11.48	24.45	1.47	42,871	240
1948	16.15	23.14	1.37	41,002	260
1949	13.95	18.12	1.44	41,560	251
1950	16.48	18.20	1.52	42,508	256
1951	20.93	20.12	1.89	46,685	282
1952	16.83	17.94	1.84	52,837	287
1953	10.67	21.65	1.82	58,320	277
1954	9.60	21.32	1.80	59,518	277
1955	10.00	14.80	1.88	61,231	276
1956	10.00	14.35	1.97	61,649	278
1957	12.06	17.89	2.03	61,146	286
1958	16.54	19.80	1.88	60,294	293

[a]Abbreviations: rec'd.-received.

[b]Includes wartime subsidies paid on beef cattle, sheep, lambs, milk, and butterfat between Oct., 1943 and June, 1946.

Table 41. Basic data used in regression equation of milk supply response for the Northeast[a]

Year	1 Tot. milk prod. on farms (mil. lb.)	2 Number of milk cows on farms (thous.)	3 Prod. per cow (lb.)	4 Milk price from comb. milk & cream marketing (dollars/cwt.)	5 Concentrate price ave. N. Atlantic states (dollars/cwt.)
1924	16,926	3,361	5,036	2.71	2.39
1925	16,753	3,293	5,087	2.90	2.42
1926	16,648	3,169	5,253	2.92	2.24
1927	16,613	3,104	5,352	3.05	2.27
1928	16,496	3,086	5.345	3.09	2.54
1929	16,495	3,093	5,333	3.17	2.49
1930	16,824	3,162	5,321	2.90	2.23
1931	17,310	3,252	5,323	2.32	1.65
1932	17,167	3,325	5,163	1.84	1.26
1933	17,093	3,356	5,093	1.90	1.37
1934	16,842	3,294	5,113	2.25	1.63
1935	16,970	3,228	5,257	2.35	1.76
1936	17,332	3,249	5,335	2.41	1.76
1937	17,434	3,247	5,369	2.47	2.04
1938	17,592	3,253	5,408	2.31	1.64
1939	17,718	3,266	5,425	2.29	1.62
1940	18,417	3,312	5,561	2.45	1.74
1941	18,946	3,351	5,654	2.74	1.88
1942	19,385	3,362	5,766	3.16	2.24
1943	18,793	3,383	5,555	3.72[b]	2.71
1944	19,162	3,456	5,545	4.35[b]	3.07
1945	19,976	3,446	5,797	4.35[b]	3.03
1946	19,303	3,367	5,733	4.95[b]	3.63
1947	19,913	3,340	5,962	5.04	4.15
1948	19,469	3,291	5,916	5.68	4.50
1949	20,905	3,294	6,346	4.82	3.66
1950	21,159	3,277	6,457	4.60	3.62
1951	21,038	3,248	6,477	5.21	4.00
1952	21,422	3,296	6,499	5.34	4.33
1953	22,385	3,379	6,625	4.90	3.94
1954	22,778	3,427	6,647	4.65	3.85
1955	23,540	3,443	6,837	4.65	3.61
1956	23,925	3,429	6,977	4.71	3.53
1957	23,486	3,333	7,059	5.03	3.60
1958	23,723	3,253	7,321	4.88	3.54

[a]Abbreviations: tot.-total; prod.-production; comb.-combined; ave.-average; mil.-million; thous.-thousand.

[b]Includes an allowance for dairy production payments Oct. 1, 1943 through June 30, 1946.

Table 42. Basic data used in regression equations of milk supply response for the Northeast.[a]

Year	1 Price beef Mid. Atlantic states (dollars/cwt.)	2 Wage ind. avg. New Eng.&Mid. Atlantic states (index)	3 Ind. prices paid for production 1910-14=100 (index)	4 Number of cattle & calves on farm Jan. 1 in U.S. (thous.)	5 Index of prices rec'd. by farmers 1910-14=100 (index)
1924	5.83	216	140	35,121	160
1925	6.22	214	145	32,315	164
1926	6.95	222	141	29,720	160
1927	7.39	224	141	27,378	159
1928	8.67	222	148	26,232	162
1929	9.15	227	146	26,975	160
1930	7.90	216	135	27,921	151
1931	5.30	182	113	29,059	130
1932	3.99	141	099	30,436	112
1933	3.89	121	099	33,420	108
1934	4.14	131	114	36,381	120
1935	7.00	134	122	32,489	124
1936	6.42	145	122	32,395	124
1937	7.75	162	132	31,245	131
1938	6.93	160	122	30,475	124
1939	6.71	162	121	30,403	123
1940	6.79	165	123	31,877	124
1941	7.81	202	130	34,372	133
1942	10.15	249	148	37,188	152
1943	12.60	305	164	40,964	171[b]
1944	11.40	343	173	44,077	182[b]
1945	13.10	369	176	44,724	190[b]
1946	14.30	405	191	43,686	208[b]
1947	17.80	436	224	42,871	240
1948	24.20	460	250	41,002	260
1949	20.30	443	238	41,560	251
1950	22.00	438	246	42,508	256
1951	28.40	491	273	46,685	282
1952	25.80	519	274	52,837	287
1953	15.00	536	256	58,320	277
1954	14.30	536	255	59,518	277
1955	14.30	551	251	61,231	276
1956	13.70	585	250	61,649	278
1957	15.10	604	257	61,146	286
1958	20.40	616	264	60,294	293

[a]Abbreviations: Mid.-Middle; ind.-index; avg.-average; Eng.-England; rec'd.-received.

[b]See footnote b, Table 40.

Table 43. Basic data used in regression equations of milk supply response for California.[a]

Year	1 Tot. milk prod. on farms (mil. lb.)	2 Number of milk cows on farms (thous.)	3 Prod. per cow (lb.)	4 Milk price from comb. milk & cream marketing (dollars/cwt.)	5 Concentrate price ave. Western states (dollars/cwt.)
1924	3,340	569	5,870	2.51	2.03
1925	3,512	573	6,130	2.66	2.14
1926	3,410	591	5,770	2.66	1.77
1927	3,660	601	6,090	2.69	1.90
1928	3,696	603	6,130	2.75	1.97
1929	3,934	610	6,450	2.77	1.93
1930	4,002	611	6,550	2.12	1.67
1931	4,052	614	6,660	1.92	1.21
1932	4,118	624	6,600	1.60	1.06
1933	4,076	630	6,470	1.57	1.04
1934	4,118	624	6,600	1.72	1.26
1935	4,151	627	6,620	1.93	1.46
1936	4,218	644	6,550	2.15	1.49
1937	4,297	656	6,550	2.31	1.75
1938	4,454	657	6,780	2.04	1.35
1939	4,617	676	6,830	1.96	1.31
1940	4,893	705	6,940	2.03	1.34
1941	5,091	740	6,880	2.43	1.54
1942	5,188	754	6,880	2.99	1.92
1943	5,222	757	6,900	3.65[b]	2.35
1944	5,479	775	7,070	4.17[b]	2.76
1945	5,720	800	7,150	4.21[b]	2.77
1946	5,866	825	7,110	4.68[b]	3.27
1947	5,974	824	7,250	4.68	3.76
1948	5,875	786	7,360	5.12	4.00
1949	5,866	780	7,520	4.52	3.35
1950	5,991	777	7,710	4.19	3.28
1951	6,014	781	7,700	4.93	3.63
1952	6,074	793	7,660	5.57	4.09
1953	6,610	816	8,100	5.16	3.78
1954	7,006	834	8,400	4.48	3.42
1955	7,242	846	8,560	4.51	3.31
1956	7,344	850	8.600	4.70	3.12
1957	7,699	866	8,390	4.79	3.21
1958	7,586	866	8,730	4.76	3.03

[a]See footnote a, Table 39 for abbreviations.

[b]See footnote b, Table 39.

Table 44. Basic data used in regression equations of milk supply response for California.[a]

Year	1 Price of beef Pacific states (dollars/cwt.)	2 Price of cows (dollars/head)	3 Number of cattle & calves on farm Jan. 1 in U.S. (thous.)	4 Index of prices rec'd by farmers 1910-14=100 (index)
1924	6.40	76	35,121	160
1925	6.80	78	32,315	164
1926	6.50	84	29,720	160
1927	7.20	89	27,378	159
1928	9.10	98	26,232	162
1929	9.40	108	26,975	160
1930	8.60	94	27,921	151
1931	5.30	64	29,059	130
1932	4.40	45	30,436	112
1933	4.25	42	33,420	108
1934	4.25	42	36,381	120
1935	7.20	66	32,489	124
1936	6.10	74	32,395	124
1937	7.20	75	31,245	131
1938	6.30	70	30,475	124
1939	7.00	70	30,403	123
1940	7.17	74	31,877	124
1941	8.18	89	34,372	133
1942	10.20	102	37,188	152
1943	12.50	129	40,964	171[b]
1944	12.00	133	44,077	182[b]
1945	12.60	133	44,724	190[b]
1946	13.70	156	43,686	208[b]
1947	18.80	175	42,871	240
1948	25.00	216	41,002	260
1949	20.00	221	41,560	251
1950	22.50	237	42,508	256
1951	28.00	288	46,685	282
1952	25.20	292	52,837	287
1953	18.10	232	58,320	277
1954	19.00	199	59,518	277
1955	18.70	191	61,231	276
1956	16.40	198	61,649	278
1957	18.80	220	61,146	286
1958	23.30	262	60,294	293

[a]Abbreviations: thous.-thousand; rec'd.-received.

[b]See footnote b, Table 40.

Table 45. Basic data used in regression equations of milk supply response for the United States.[a]

Year	1 Tot. milk prod. on farms (mil. lb.)	2 Number of milk cows on farms (thous.)	3 Prod. per cow (lb.)	4 Milk price from comb. milk & cream marketing (dollars/cwt.)	5 Concentrate price ave. United States (dollars/cwt.)
1924	89,240	21,417	4,167	2.16	1.90
1925	90,699	21,503	4,218	2.26	1.92
1926	93,325	21,312	4,379	2.25	1.67
1927	95,172	21,191	4,491	2.35	1.75
1928	95,843	21,223	4,516	2.41	1.97
1929	98,988	21,618	4,579	2.39	1.88
1930	100,158	22,218	4,508	2.07	1.65
1931	103,029	23,108	4,459	1.60	1.15
1932	103,810	24,105	4,307	1.24	.86
1933	104,762	25,062	4,180	1.26	.97
1934	101,621	25,198	4,033	1.49	1.34
1935	101,205	24,187	4,184	1.69	1.48
1936	102,410	23,727	4,316	1.86	1.47
1937	101,908	23,340	4,366	1.92	1.74
1938	105,807	23,215	4,558	1.66	1.22
1939	106,792	23,273	4,589	1.59	1.22
1940	109,412	23,671	4,622	1.74	1.34
1941	115,088	24,288	4,738	2.04	1.48
1942	118,533	25,027	4,736	2.40	1.85
1943	117,017	25,451	4,598	2.93	2.28
1944	117,023	25,597	4,572	3.45	2.61
1945	119,828	25,033	4,787	3.50	2.57
1946	117,697	24,089	4,886	4.06	3.06
1947	116,814	23,329	5,007	4.12	3.63
1948	112,671	22,336	5,044	4.66	3.84
1949	116,103	22,024	5,272	3.81	3.02
1950	116,602	21,944	5,314	3.75	3.08
1951	114,681	21,505	5,333	4.40	3.52
1952	114,671	21,338	5,374	4.68	3.75
1953	120,221	21,691	5,542	4.19	3.43
1954	122,094	21,581	5,657	3.86	3.30
1955	123,128	21,193	5,810	3.89	3.10
1956	125,698	20,900	6,004	4.02	3.00
1957	125,939	20,443	6,160	4.12	3.00
1958	125,236	19,784	6,330	4.04	2.89

[a]Abbreviations: see footnote a, Table 39.

Table 46. Basic data used in regression equations of milk supply response for the United States.

Year	1 Index prices rec'd. for meat animals 1910-14=100 (index)	2 Price cutter & canner cows Chicago (dollars/cwt.)	3 Price all packer and shipper hogs Chicago (dollars/cwt.)	4 Index of prices rec'd. by farmers 1910-14=100 (index)
1924	109	3.08	8.11	160
1925	139	3.48	11.81	164
1926	146	4.25	12.34	160
1927	138	5.11	9.95	159
1928	150	6.49	9.22	162
1929	155	6.32	10.16	160
1930	133	4.49	9.47	151
1931	91	2.96	6.16	130
1932	63	2.07	3.83	112
1933	59	2.01	3.94	108
1934	68	2.05	4.65	120
1935	115	3.78	9.27	124
1936	118	4.04	9.89	124
1937	130	4.54	10.02	131
1938	113	4.63	8.09	124
1939	110	4.84	6.57	123
1940	108	4.65	5.71	124
1941	143	5.68	9.45	133
1942	186	7.22	13.70	152
1943	203	8.18	14.31	171[b]
1944	190	7.32	13.57	182[b]
1945	207[a]	8.30	14.66	190[b]
1946	248[a]	9.12	18.40	208[b]
1947	329	11.48	24.45	240
1948	361	16.15	23.14	260
1949	311	13.95	18.12	251
1950	340	16.48	18.20	256
1951	409	20.93	20.12	282
1952	353	16.83	17.94	287
1953	288	10.67	21.65	277
1954	283	9.60	21.32	277
1955	246	10.00	14.80	276
1956	235	10.00	14.35	278
1957	275	12.06	17.89	286
1958	334	16.54	19.80	293

[a]Includes wartime subsidies paid on beef cattle, sheep, lambs between July, 1945 and June, 1946.

[b]See footnote b, Table 40.

Table 47. Bibliographic references for the time series in Appendix A by table and column number.

Table	Column number	Bibliographic reference
39	1	64, 70
	2	64, 70
	3	64, 70
	4	64, 70
	5	64, 70
40	1	67, 71
	2	67, 71
	3	66, 73
	4	67, 71
	5	58, 59
41	1	64, 70
	2	64, 70
	3	64, 70
	4	64, 70
	5	64, 70
42	1	60, 62
	2	73, 74
	3	58, 59
	4	67, 71
	5	58, 59
43	1	64, 70
	2	64, 70
	3	64, 70
	4	64, 70
	5	64, 70
44	1	60, 62
	2	64, 70
	3	67, 71
	4	58, 59
45	1	64, 70
	2	64, 70
	3	64, 70
	4	64, 70
	5	64, 70
46	1	58, 59
	2	67, 71
	3	67, 71
	4	58, 59

APPENDIX B

The long-run coefficient of adjustment is derived below. The equations used are adapted from Koyck (40, p. 20-23). The relationship between the difference equation of Koyck and the model employed by Nerlove (50) is also shown.

The concept of a distributed lag can be expressed as follows:

(100) $$Y_t = \alpha_1 P_{t-1} + \alpha_2 P_{t-2} \ldots \alpha_n P_{t-n}$$

where:

Y_t = the output in the current year

P_{t-1} = the price in the previous year

α_i = the coefficient for price

This equation states that present output is a function of past prices. The long-term reaction is given by $\sum \alpha_i$. For data computed in logarithms this summation is equivalent to the long-run elasticity. If original data are used, the long-run elasticity is $(\sum \alpha_i)(P/Y)$ where P and Y are designated values (e.g., mean values).

However, calculation of the coefficients of a variable lagged several time periods is not statistically possible. All or a portion of the time path of the lag can be approximated by a converging geometric series. The following relationship is assumed for the coefficients of the converging series:

(101) $$\alpha_{k+m} = \lambda \alpha_{k+m-1} \qquad m \geq 0 \qquad 0 \leq \lambda < 1$$

The subscripts of the coefficients indicate the time period. The point at which the series begins is designated by k (i.e., the adjustment path is approximated from k onward). If the entire path of the adjustment is approximated by the converging geometric series the equation is:

(102) $$Y_t = \alpha_1 P_{t-1} + \alpha_1 \lambda P_{t-2} + \alpha_1 \lambda^2 P_{t-3}$$

The higher the value of λ the more gradually the coefficients of successive lagged variables will decline. Full adjustment will take place over a longer period of time. A second equation may be written:

(103) $$\lambda Y_{t-1} = \alpha_1 \lambda P_{t-2} + \alpha_1 \lambda^2 + P_{t-3} \ldots$$

The values of successive coefficients of both Equations 102 and 103 approach zero. Therefore, subtracting Equation 102 from 103 gives:

(104) $$Y_t - \lambda Y_{t-1} = \alpha_1 P_{t-1} \quad \text{or}$$

(105) $$Y_t - Y_{t-1} = \alpha_1 P_{t-1} - (1-\lambda) Y_{t-1}$$

where:

$(1-\lambda) = \beta$, the coefficient of expectation

The higher the value of β, the more rapid the adjustment. The long-run reaction is equal to $\sum_{i=0}^{\infty} \alpha_1 \lambda^i$, the sum of the coefficients of the geometric progression. This in turn is equivalent to:

(106) $$\sum_{i=0}^{\infty} \alpha_1 \lambda^i = \frac{\alpha_1}{1-\lambda} = \frac{\alpha_1}{\beta}$$

Hence, the coefficient of long-run reaction is the coefficient of lagged price divided by the coefficient of lagged output.

Nerlove has added Y_{t-1} to both sides of Equation 105 to obtain the relation:

(107) $$Y_t = \alpha_1 P_{t-1} - (1-\beta) Y_{t-1}$$

APPENDIX C

A variable moving average for both hog price and the milk-feed price ratio has been used in several equations of the analysis. These moving averages were calculated in the following manner. First, the observations were plotted. Then mid-points were established between the peak and trough of each cycle. This is illustrated in Figure 5 for the hog cycle. A dotted line joins these midpoints and approximates the moving average. In computing the series used in this study, an average was taken centered on each year. The number of years entering each average was based upon the length of the cycle at that point. For example, there is a period of nine years from trough to trough in the hog series between 1932 and 1940. This span represents a single cycle. The value for 1936, the middle year, is determined by taking the nine year average.

The number of years averaged will vary just as the length of the cycles vary. The midpoint between the trough at 1932 and the peak at 1937 is at 1934 and the midpoint between the trough at 1940 and the following peak at 1943 is at 1941. The length of the cycle from midpoint to midpoint is only eight years and is centered between 1937 and 1938. The value for 1937 is the average of the years from 1935 to 1940 plus one-half of 1934 and 1941. The length of the cycle was determined to the nearest year and in the case of hogs varied from a high of nine years to a low of three years. For a more detailed discussion of smoothing procedures see Maverick (43).

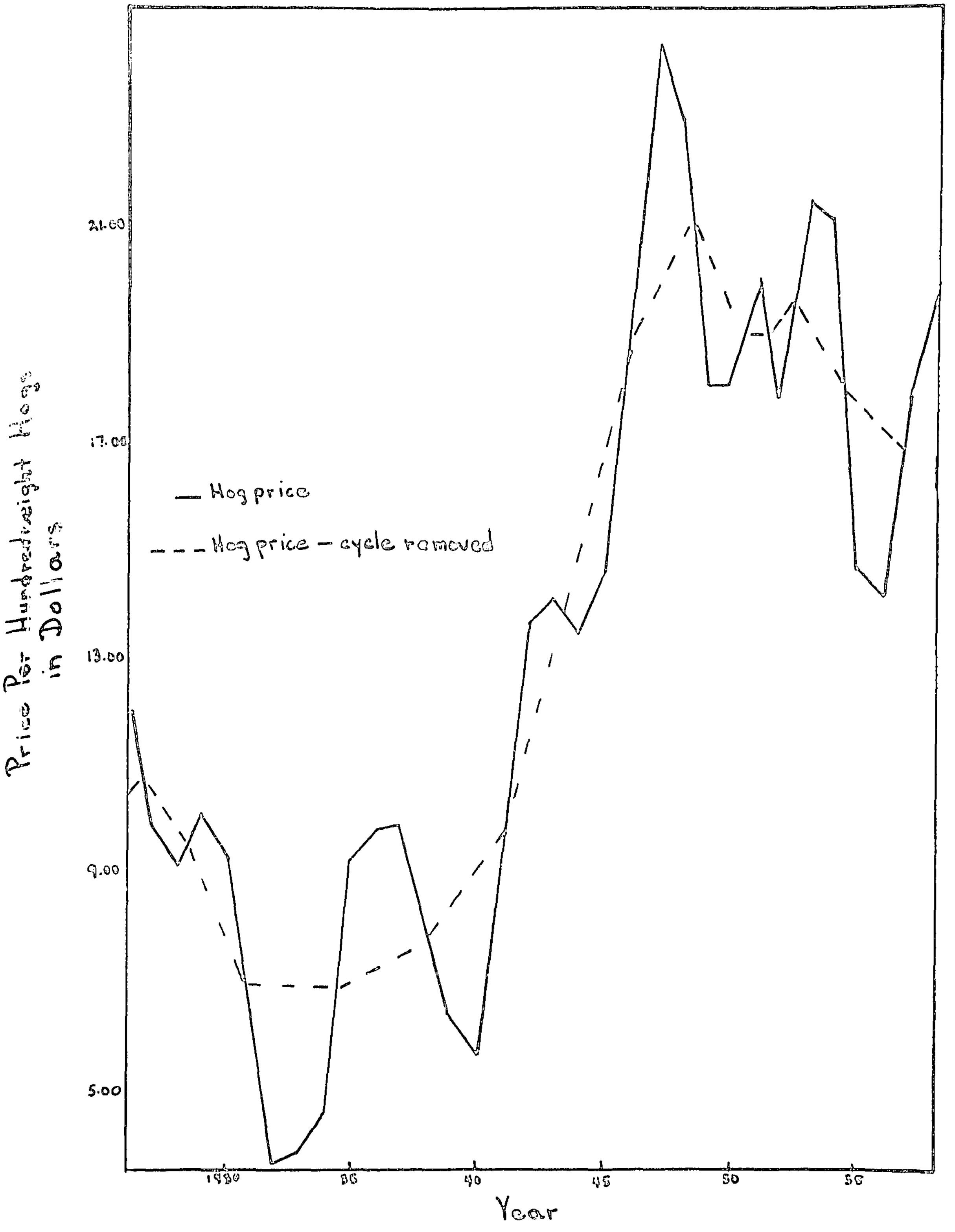

Figure 5. Price per cwt. of hogs, Chicago: showing approximation of a variable moving average to remove the effect of hog cycles.

APPENDIX D

Short-run changes in production per cow are strongly influenced by the quantity and quality of the ration fed. It is particularly difficult, however, when working with annual data to select a variable which will adequately reflect this change. The problem is illustrated in Tables 48 and 49. These tables show the simple correlations between production per cow, grain fed per cow, forage fed, hay quality, and pasture conditions. New York State data for winter and summer feeding periods 1939-58 were used because a series measuring hay quality during those years was available.

Information for constructing the tables was obtained from New York State Dairy Farm Reports (52). Figures on production per cow and grain fed per cow are received from farm reporters. Quantity of forage fed represents the quantity of hay and silage produced plus the change in hay inventory. This figure was considered to be a realistic measure in New York where nearly all forage is fed to dairy cows. Quality of forage is assumed to be reflected by farm reports of the percentage of the hay crop harvested by August 1 each year. Late harvested hay is of lower quality. Correlations were computed using first differences of logarithms to remove the trend.

Table 48. Correlations, New York State data for summer pasture seasons, 1939-51: showing correlation coefficients for production per cow, the quantity of grain fed per cow, and pasture conditions.

	Production per cow	Quantity grain fed	Pasture conditions
Production per cow	1		
Grain per cow	.252	1	
Pasture conditions	.617	-.413	1

Table 49. Correlations, New York State data for winter feeding period, 1939-40 to 1957-58: showing correlation coefficients for production per cow, the quantity of grain fed per cow, the quantity of forage fed, the hay quality and pasture conditions.[a]

	Prod. per cow	Quantity grain fed	Quantity forage fed	Hay quality	Pasture conditions
Prod. per cow	1				
Quantity grain fed	.540	1			
Quantity forage fed	.046	.229	1		
Hay quality	.755	.602	.038	1	
Pasture conditions	-.655	-.408	.005	-.624	1

[a]Abbreviation: Prod.-production.

Milk production per cow has a correlation of .617 with pasture conditions during the summer pasture season. The negative correlation between grain fed per cow and pasture conditions (-.413) indicates that farmers increase grain rations when pasture is in poor condition. This substitution mitigates the effect of drought on milk production.

Production per cow is negatively correlated with pasture conditions for the previous season (-.655) and positively correlated with hay quality (.755). Years of good pasture and high summer milk production are accompanied by years of poor hay quality and low winter production. This is because an abundant rainfall not only damages hay, but delays hay harvesting operations until much of the crop is well past matur-

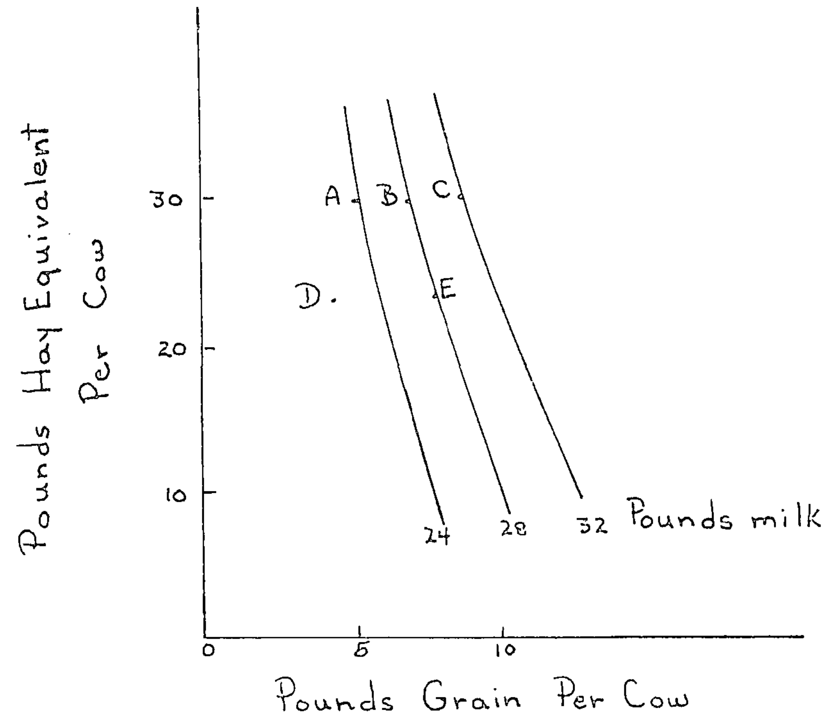

Figure 6. Isoquants showing substitution rates of grain and forage for indicated average daily production per cow.

ity at the time of harvesting. Under this assumption the New York market administrator has successfully used May rainfall as a variable in predicting November milk production.

The positive correlation between hay quality and grain fed per cow (.602) indicates that farmers feed in the winter according to milk production per cow. When poor quality hay lowers production farmers feed less grain. The physical quantity of forage fed appears to have little influence on milk production. The correlation is close to zero. In most years roughage is fed in liberal quantities at close to stomach capacity. The quantity fed does not vary to any great degree except in years when hay is in very short supply. Figure 6 illustrates this situation hypothetically. Assume that under normal conditions a cow is fed 30 pounds of hay equivalent per day with seven pounds of grain and produces 28 pounds of milk. Grain is fed at a ratio of four to one (i.e., one pound of grain for every four pounds of milk produced). When the quality of hay improves, production increases and the quantity of grain will be increased in accordance with the ratio. The shift to point C on the new isoquant represents an increase in production to 32 pounds. The slope of this isoquant shifts. The higher quality forage now substitutes for less grain to maintain a constant production. In years of ample hay supply farmers will feed along the line running from A to C. The line rises slightly since cows will tend to eat more of the high quality forage. However, in years of shortage movement from point B will be in the direction of either D or E. If possible, movement will be along the isoquant as farmers substitute grain for forage in the ration. On the other hand, if grain is also in short supply movement will be in the direction of point D. This was the situation in 1934 in the Lake States when short supplies of grain and forage brought a sharp decline in production per cow. By contrast, a short supply of roughage in 1949 was accompanied by one of the largest supplies of grain on record. Feed prices were low and production per cow increased by approximately 400 pounds. In general, it appears that movement toward point D was more typical during droughts of the depression years, and that movement toward point E is more typical of postwar years when grain has been in surplus. However, in most years when there is adequate roughage available, quality is undoubtedly more important than quantity.

The hypothesis advanced above concerning farmer response during summer and winter feeding period is supported by the results of earlier studies conducted by Halvorson (31) and Brandow (5).

APPENDIX E

The table below shows the grouping of years according to milk-feed price movements in the long and short run. Regressions were run for observations in each of these groups to test the hypothesis that the supply function is irreversible.

Table 50. The grouping of years according to price movements for the statistical analysis of the reversibility of the milk supply function for the Lake States, the Northeast, and California.

Direction of price movement	Lake States		Northeast		California	
Long-run rising	1926	1944	1926	1942	1926	1945
Short-run rising	1927	1946	1927	1944	1927	1946
	1938	1952	1929	1945	1930	1952
	1940	1955	1930	1952	1931	1953
	1941	1956	1931	1953	1932	1954
	1943		1932	1955	1940	1956
			1940	1956	1941	1957
			1941		1942	1958
					1943	
Long-run rising	1937	1953	1928	1954	1928	1944
Short-run falling	1939	1954	1943	1957	1929	1955
	1942	1957	1946	1958		
	1945	1958				
	1947					
Long-run falling	1928	1934	1933	1937	1933	1938
Short-run falling	1929	1935	1934	1947	1934	1947
	1930	1949	1935	1949	1935	1948
	1933	1950	1936	1950	1936	1951
Long-run falling	1931	1948	1938	1948	1937	1949
Short-run rising	1932	1951	1939	1951	1939	1950
	1936					

APPENDIX F

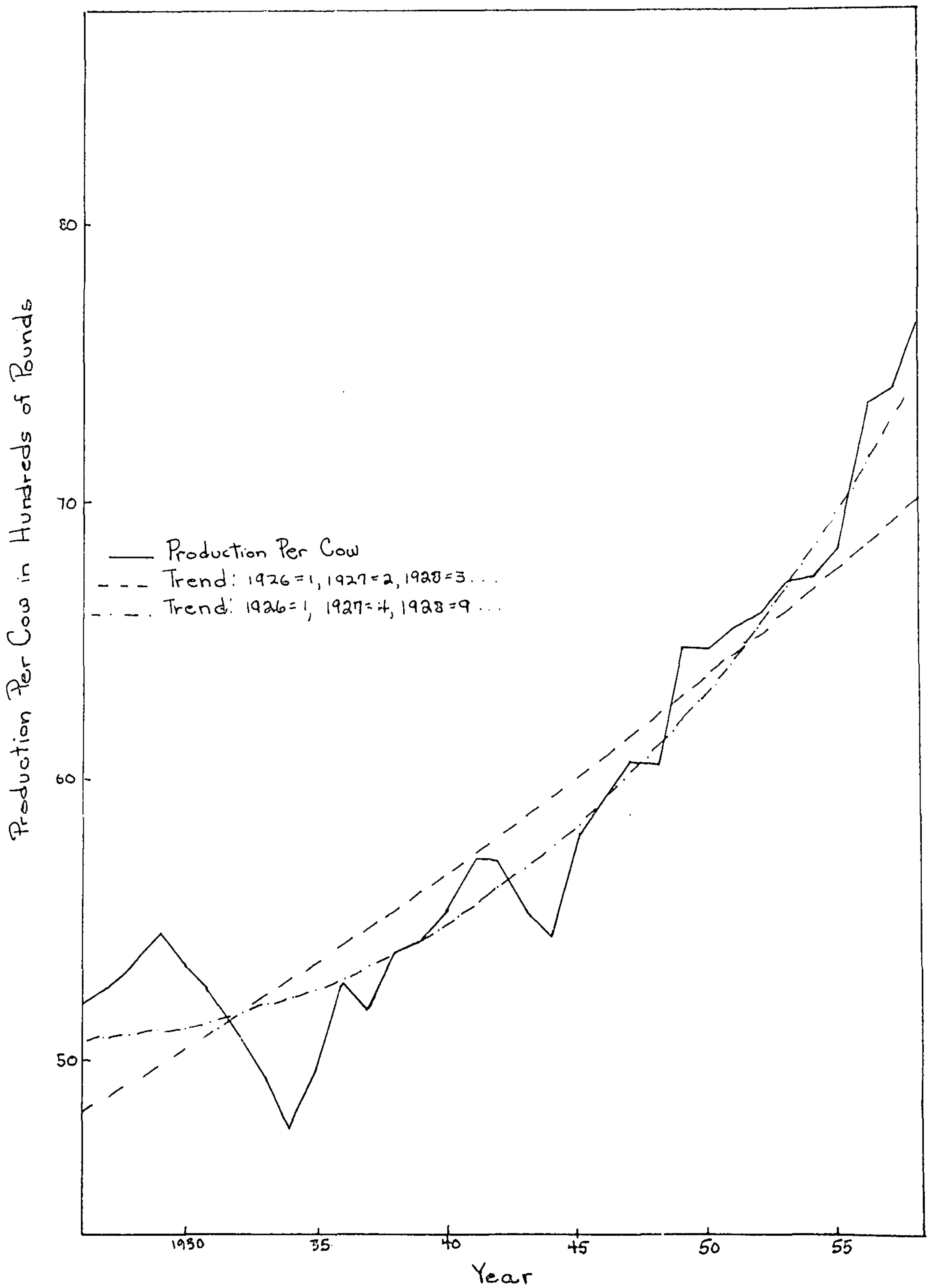

Figure 7. Production per cow, Lake States, 1926-58, exponential trends fitted to production data for time: 1926 = 1, 1927 = 2...and for squared values of time: 1926 = 1, 1927 = 4...

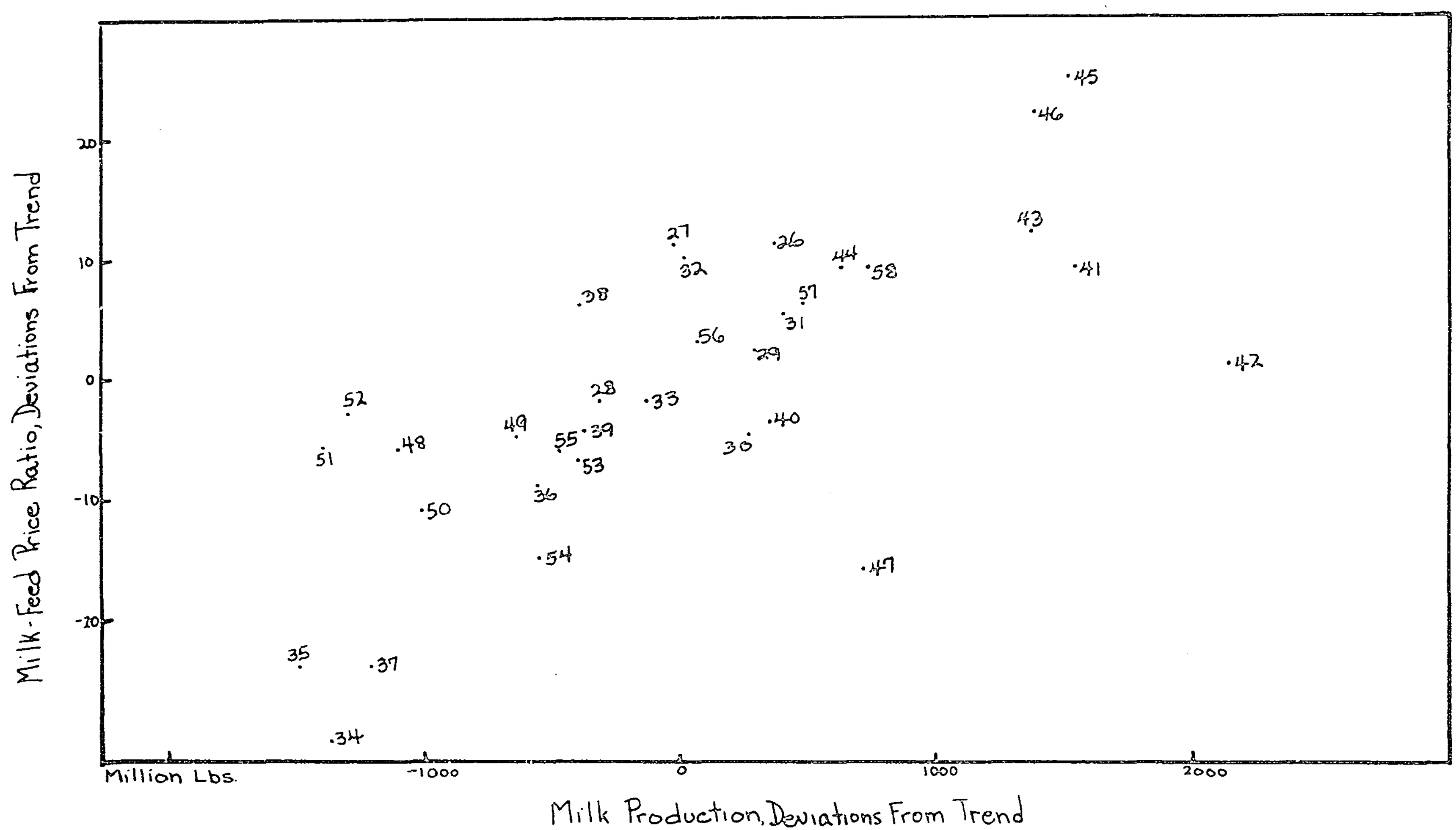

Figure 8. Annual observations of milk-feed price ratio vs. milk production, Lake States, 1926-58: observations for each series are deviations from the exponential trend: 1926 = 1, 1927 = 2...

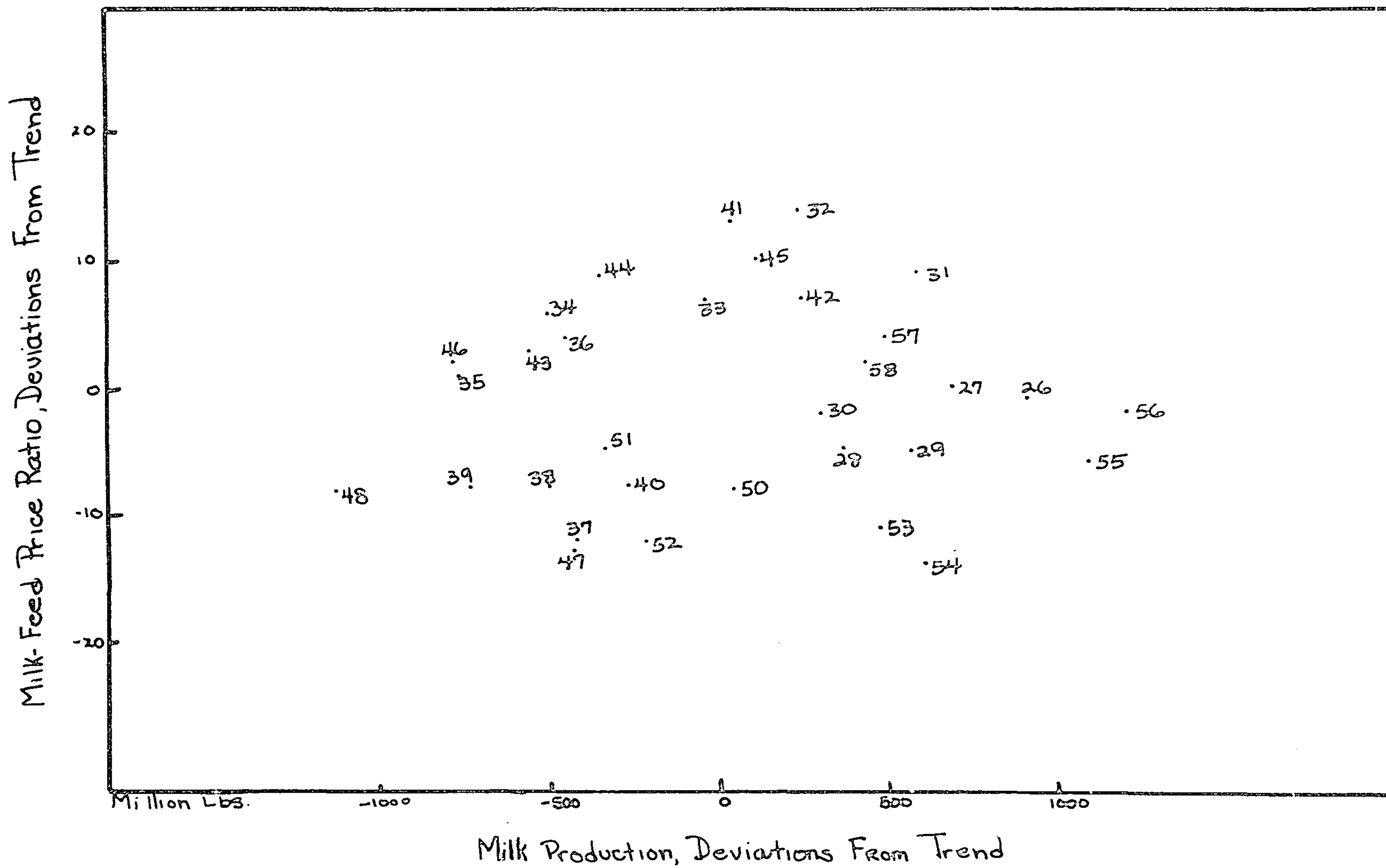

Figure 9. Annual observations, milk-feed price ratio vs. milk production, Northeast, 1926-58: observations for each series are deviations from the exponential trend: 1926 = 1, 1927 = 2...

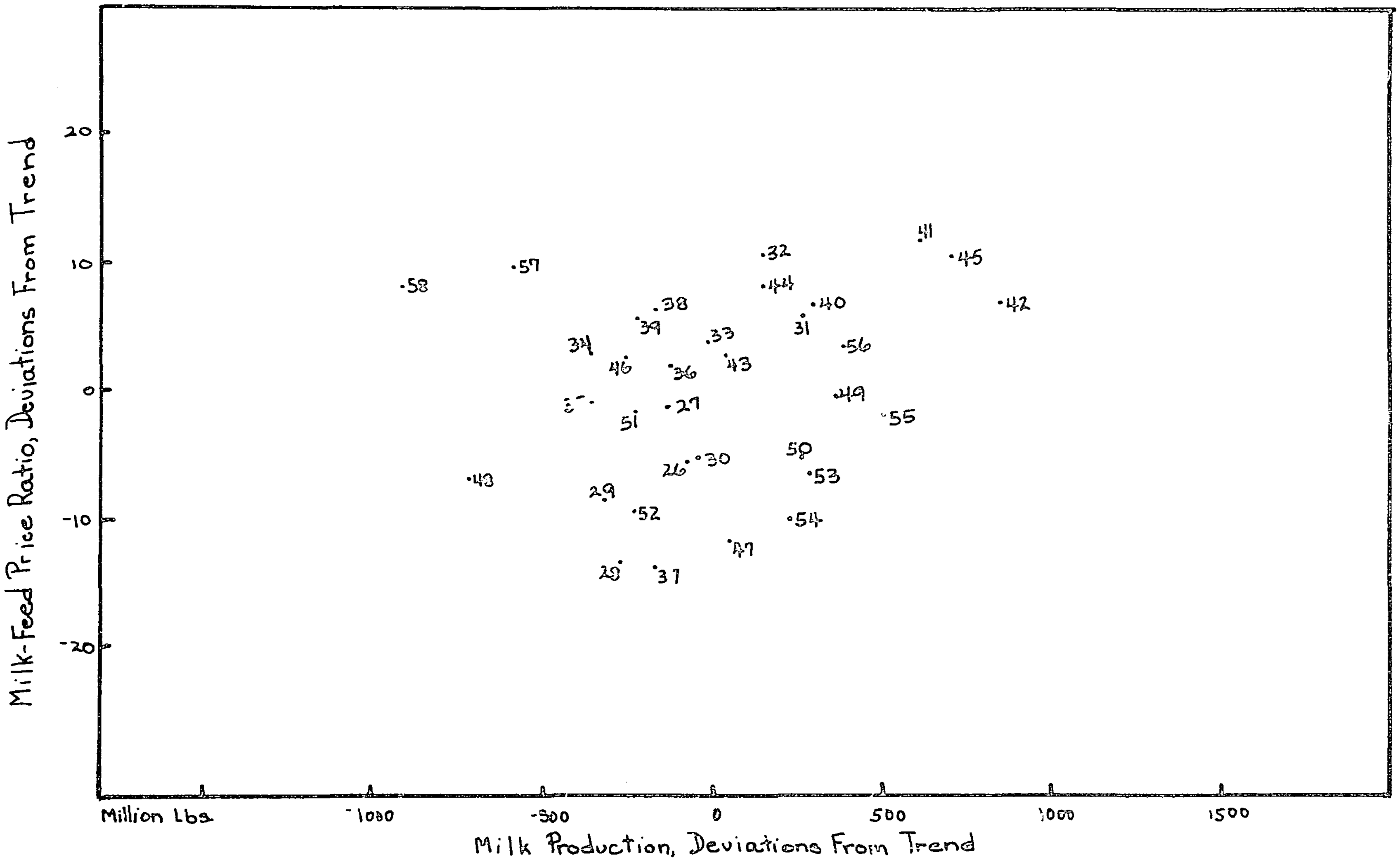

Figure 10. Annual observations, milk-feed price ratio vs. milk production, Northeast, 1926-58. Observations for each series are deviations from squared values of the trend: 1926 = 1, 1927 = 4...

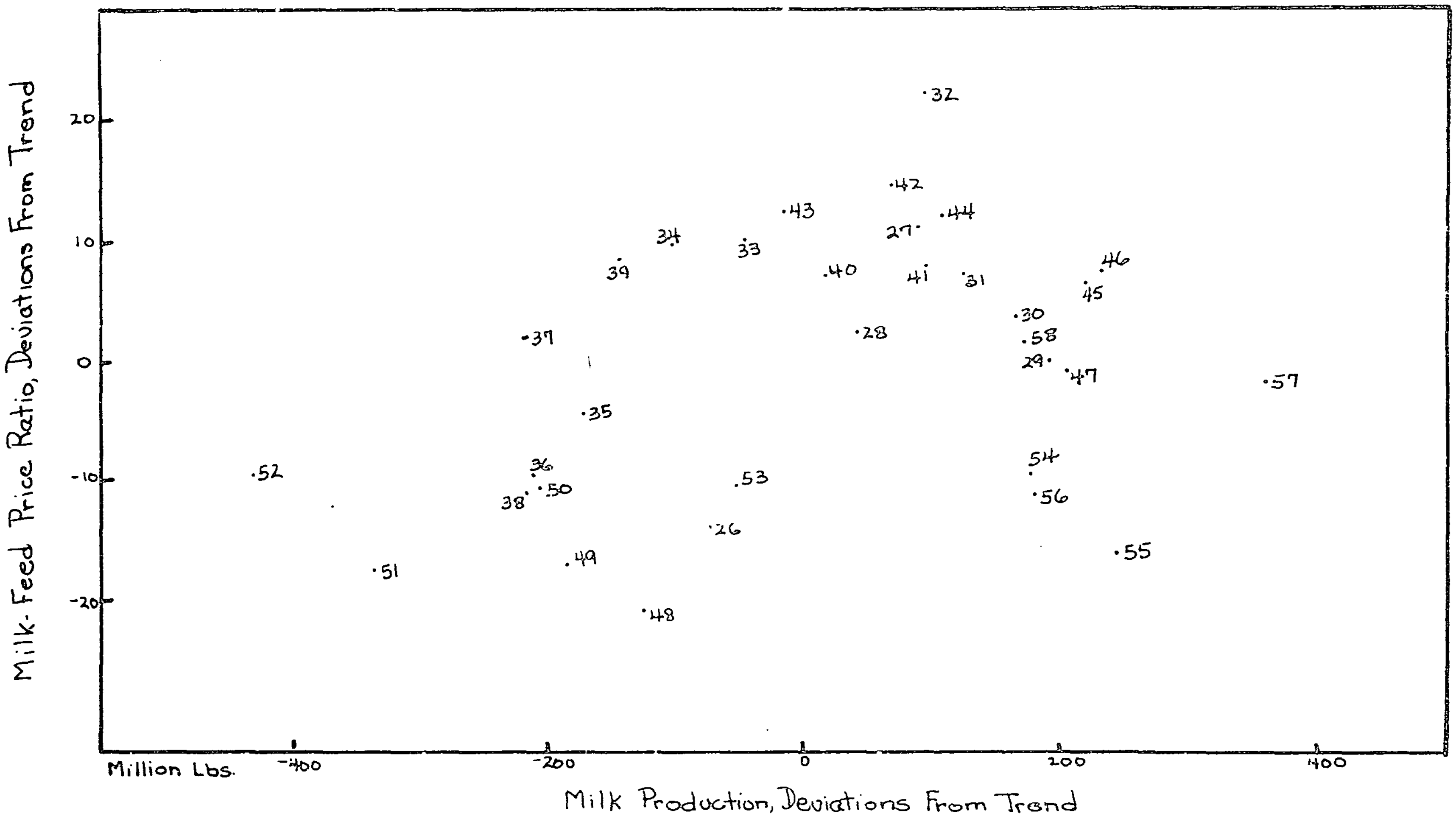

Figure 11. Annual observations, milk-feed price ratio vs. milk production, California, 1926-58. Observations for the series are deviations from the exponential trend: 1926 = 1, 1927 = 2...